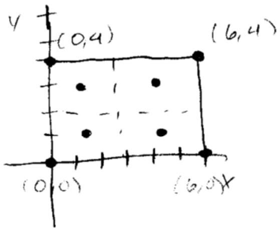
Exam 2 Calc 3 10/21/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers. All integrals should be set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no x or y, etc.

10

1. Write a double Riemann sum for $\iint_R f\,dA$, where R is the rectangle with vertices (0,0), (6,0), (6,4), (0,4) using midpoints with $n = m = 2$ subdivisions.



midpoints = $(1.5,1), (4.5,1), (1.5,3), (4.5,3)$

$$\iint_R f\,dA \approx 6f(1.5,1) + 6f(4.5,1) + 6f(1.5,3) + 6f(4.5,3)$$

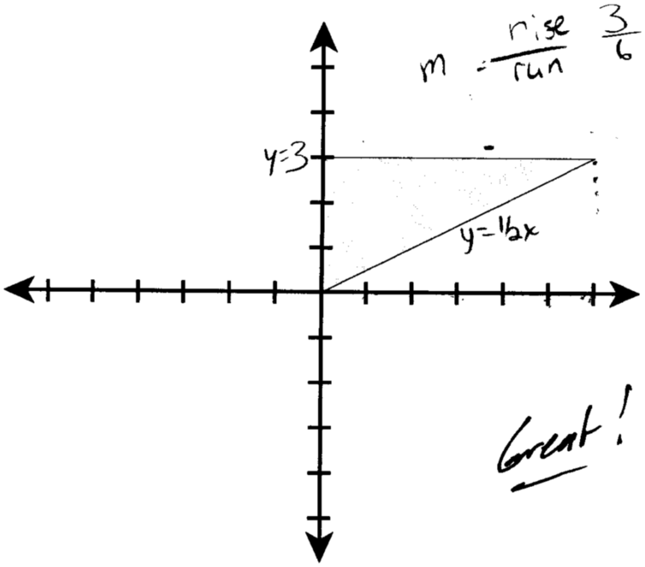
$\Delta A = \Delta x \Delta y = 3 \cdot 2 = 6$

Good

2. Set up an iterated integral for the volume below $z = f(x, y)$ and above the xy-plane on the region R pictured below:

$m = \frac{\text{rise}}{\text{run}} \; \frac{3}{6} = 1/2$

$y = 1/2x$



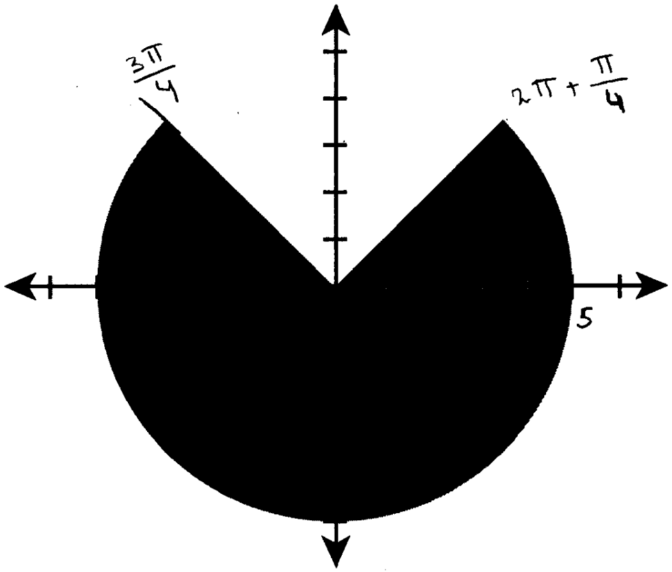
$$\int_0^6 \int_{1/2x}^{3} \int_0^{f(x,y)} dz\,dy\,dx$$

or

$$\int_0^6 \int_{1/2x}^{3} f(x,y)\,dy\,dx$$

Great!

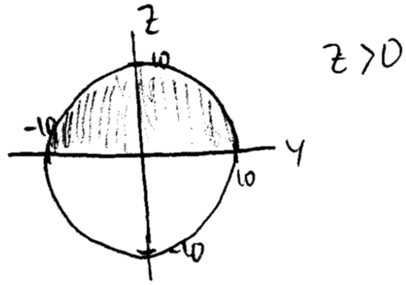
3. Set up an iterated integral for the total mass of a plate shaped like the region shown below, with density $\rho(x, y) = k$.



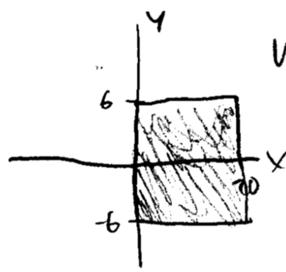
$$\int_{\frac{3\pi}{4}}^{2\pi+\frac{\pi}{4}} \int_0^5 k\, r\, dr\, d\theta$$

Good!

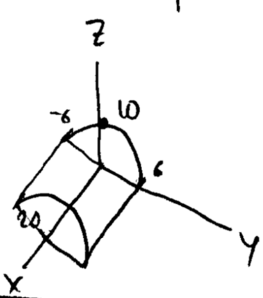
4. Set up an iterated integral for the volume of a really nice greenhouse with a roof shaped like $y^2 + z^2 = 100$ and base a rectangle $R = [0,20] \times [-6,6]$.



$z > 0$



not to scale



$$\boxed{V = \int_0^{20} \int_{-6}^{6} \int_0^{\sqrt{100-y^2}} 1 \, dz\, dy\, dx}$$

Great

5. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} 3\,dz\,dy\,dx$.

$$= 3\cdot \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} 1\,dz\,dy\,dx$$

$= 3 \cdot$ The volume of one quarter of a sphere with $r=1$

$$= 3\cdot \frac{4}{3}\pi(1)^3\cdot\frac{1}{4}$$

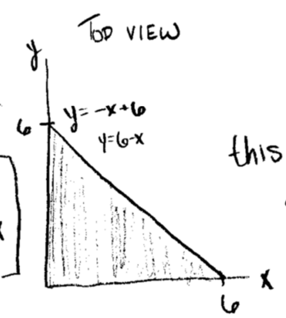
$$= \pi$$

6. The waiting time for certain support phone calls includes an initial hold period of length x and a subsequent hold period of length y once the call is directed to an appropriate specialist, for the random variables X and Y with joint density function

$$p(x,y)=\begin{cases}\frac{1}{15}e^{-x/3-y/5} & \text{for } x\ge 0, y\ge 0\\ 0 & \text{otherwise}\end{cases}$$

Set up an iterated integral for the probability that total hold time for a call is over 6 minutes.

probability for more than 6 minutes $= 1-\left[\int_0^6\int_0^{6-x}\frac{1}{15}e^{-\frac{x}{3}-\frac{y}{5}}\,dy\,dx\right]$



this is probability it takes less than 6 minutes so 1 - shaded area will find desired probability

Great.

8. Find the Jacobian for the transformation from rectangular to spherical coordinates:

$x = \rho \sin \phi \cos \theta$
$y = \rho \sin \phi \sin \theta$
$z = \rho \cos \phi$

$$\begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} & \frac{\partial z}{\partial \phi} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \end{vmatrix}$$

$$\begin{vmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & \cos\phi \\ p\cos\phi\cos\theta & p\cos\phi\sin\theta & -p\sin\phi \\ -p\sin\phi\sin\theta & p\sin\phi\cos\theta & 0 \end{vmatrix}$$

$$(0 + \underline{p^2 \sin\phi \cos^2\theta \cos^2\phi} + \underline{p^2 \sin^3\phi \sin^2\theta}) - (-\underline{p^2 \sin\phi \sin^2\theta \cos^2\phi} + -\underline{p^2 \sin^3\phi \cos^2\theta} + 0)$$

$$p^2 \sin\phi (\cos^2\theta \cos^2\phi + \sin^2\phi \sin^2\theta + \sin^2\theta \cos^2\phi + \sin^2\phi \cos^2\theta)$$

$$p^2 \sin\phi (\cos^2\theta (\underbrace{\cos^2\phi + \sin^2\phi}_{1}) + \sin^2\theta (\underbrace{\cos^2\phi + \sin^2\phi}_{1}))$$

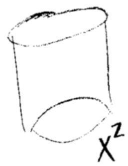
$$p^2 \sin\phi (\underbrace{\cos^2\theta + \sin^2\theta}_{1})$$

Great

1/7 $\boxed{p^2 \sin\phi}$

9. A hole with radius 2cm is drilled vertically through a sphere with radius 10cm. The sphere is made of copper, which has a density of 8.96 g/cm³. Set up an iterated integral for the mass of the resulting solid.

Cylindrical



$x^2+y^2=4$

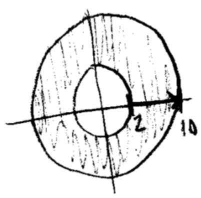
$x^2+y^2+z^2=10^2 \quad r^2+z^2=10^2$

$z=\pm\sqrt{10^2-r^2}$

$$M=\int_0^{2\pi}\int_2^{10}\int_{-\sqrt{10^2-r^2}}^{\sqrt{10^2-r^2}} 8.96\, r \; dz\, dr\, d\theta$$

Excellent!

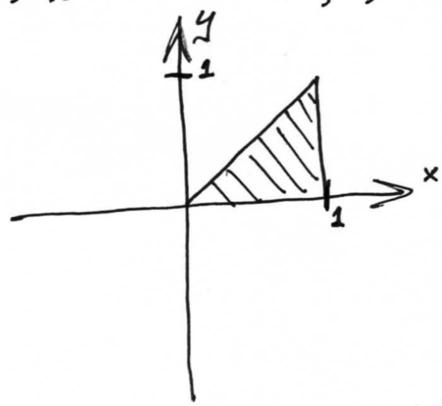
TOP VIEW



$\leftarrow 0 \le \theta \le 2\pi$

10. Set up iterated integrals for the z coordinate of the center of mass of a pyramid with uniform density, height 1, and a square base with side length 2.

It seems easiest to position the pyramid so its base is the square $[-1,1]\times[-1,1]$, and its vertex is at $(0,0,1)$. Then let's just deal with the part shown at right, since its center of mass will be the same as the other seven symmetric copies.



The "roof" on this section is $z=1-x$, so our integrals are:

$$\bar{z} = \frac{\int_0^1 \int_0^x \int_0^{1-x} k\cdot z \, dz\,dy\,dx}{\int_0^1 \int_0^x \int_0^{1-x} k \, dz\,dy\,dx}$$