Exam 3 Calculus 3 12/2/2016

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

$$\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$
$$\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$x = \rho \sin \phi \cos \theta$
$y = \rho \sin \phi \sin \theta$
$z = \rho \cos \phi$

1. Parametrize and give bounds for a counterclockwise circle, centered at the origin, with radius 5, beginning and ending at (5,0).

2. Evaluate $\int_C \left(x^2 - y\right) dx + x\,dy$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

3. Let $\mathbf{F}(x, y, z) = \langle 3y, 2x + 5y - z, x^2 + 2z \rangle$. Evaluate div $\mathbf{F}$.

4. Let $\mathbf{F} = \langle -\sin x \cos y, -\cos x \sin y \rangle$, and let C be a line segment from $(0, 0)$ to $(2\pi, \pi)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

5. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} - x\,\mathbf{k}$ and C is the line segment from (2, –3, 0) to (0, 0, 7).

6. Prove that if **F**(x,y,z) is a vector field whose component functions have continuous second-order partial derivatives, then div(curl **F**) = 0. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calc 3 student at a large state university and she's having some trouble. Bunny says "Ohmygod, I totally hate true/false math test questions! There was this one, it said a circulation in a conservative vector field is *always* zero. I marked it false because I don't think very many things are *always* true, you know? But it turned out it is. What does that mean, anyway?"

   Explain as clearly as possible to Bunny how she could know that a circulation in a conservative vector field is *always* zero.

8. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle 0, 0, z^3/3 \rangle$ and S is the sphere $x^2 + y^2 + z^2 = 1$.

9. Let **F** be the vector field $\mathbf{F} = y^2\,\mathbf{i} + xz\,\mathbf{j} - x\,\mathbf{k}$. Let S be the portion of the paraboloid $y = x^2 + z^2$ to the left of $y = 36$, with normal vectors oriented in the positive y direction. Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

10. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle x, y, 2z \rangle$ and S is the portion of the plane $z = 3$ that lies within the cylinder with equation $x^2 + y^2 = 9$, oriented upwards.

Extra Credit (5 points possible): Let $\mathbf{F}(x, y, z) = \langle x/r^3, y/r^3, z/r^3 \rangle$, where here $r = \sqrt{x^2 + y^2 + z^2}$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is a sphere with radius 1 centered at the point (a, b, c).