

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 3 points for correct and clearly justified answers, and spelling your name correctly on your submission is worth 1 point.

1. [Based on Rogawski & Adams 3rd §15.5 Example 7] Without proper maintenance, the times to failure (in months) of two sensors in an aircraft are random variables X and Y with joint density function

$$p(x, y) = \begin{cases} \frac{1}{864} e^{-x/24 - y/36} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that neither sensor functions after 1 year?

2. [Based on Rogawski & Adams 3rd §15.5 #51] The lifetime (in months) of two components in a certain device are random variables X and Y with joint density function

$$p(x, y) = \begin{cases} \frac{1}{9216} (48 - 2x - y) & \text{for } x \geq 0, y \geq 0, 2x + y \leq 48 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the probability that both components function at least 6 months without failing.

3. The waiting time for certain support phone calls includes an initial hold period of length x and a subsequent hold period of length y once the call is directed to an appropriate specialist, for the random variables X and Y with joint density function

$$p(x, y) = \begin{cases} \frac{1}{15} e^{-x/3 - y/5} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the total hold time for such a call is over 10 minutes?