Exam 2b **Real Analysis 1** **11/3/16**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. a) State the definition of the derivative of a function $f(x)$ at $x = a$.

   b) State the definition of continuity of a function $f(x)$ at $x = a$.

2. a) State the definition of a set E being closed.

b) State the definition of a set E being open.

3. a) State the Intermediate Value Theorem.

b) State Fermat's Theorem.

4. a) State the definition of a compact set.

b) State the Heine-Borel Theorem.

c) Give an example of a set with an open cover that has no finite subcover.

5. a) State the definition of uniform continuity.

b) Give an example of a function which is continuous at every real number, but not uniformly continuous on $\mathbb{R}$.

6. State and prove the Product Rule for Derivatives.

7. State and prove the Mean Value Theorem.

8. State and prove the Extreme Value Theorem.

9. a) Prove or give a counterexample: If f' is bounded, then f is bounded.

b) Prove or give a counterexample: If f is bounded, then f' is bounded.

10. a) Prove or give a counterexample: If a function is defined on $[a, b]$ and continuous on (a, b), then it attains its maximum and minimum values on $[a, b]$.

b) Prove or give a counterexample: If a function is continuous and bounded on $\mathbb{R}$, then it attains its maximum and minimum values.