Problem Set 3 **Real Analysis 1** **Due 9/14/16**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. If a sequence $\{a_n\}$ converges to 0, and a sequence $\{b_n\}$ is bounded, then the sequence $\{a_n b_n\}$ converges to 0.

2. If a sequence $\{a_n\}$ converges to 0, and $\{a_n b_n\}$ converges to zero, then the sequence $\{b_n\}$ is bounded.

3. Determine whether $\lim_{n\to\infty} \frac{1}{n} \sin \frac{1}{n}$ exists, and find its value. [Kosmala 2.2.11(l)]

4. Consider the sequences $\{a_n\}$ and $\{b_n\}$, where sequence $\{a_n\}$ converges to zero. Then $\{a_n b_n\}$ converges to zero. [Kosmala 2.2.12]

5. If a sequence $\{a_n\}$ diverges to $+\infty$ and $a_n \leq b_n$ for all $n \geq n_1$, then the sequence $\{b_n\}$ must also diverge to $+\infty$.

6. Give an example of an oscillating sequence that is: [Kosmala 2.3.4]
   (a) bounded.
   (b) bounded above and unbounded below.
   (c) bounded below and unbounded above.
   (d) unbounded below and unbounded above.

7. If sequences $\{a_n\}$ and $\{b_n\}$ diverge to $+\infty$, then $\{a_n b_n\}$ diverges to $+\infty$. [Kosmala 2.3.5]

8. Suppose that the sequence $\{a_n\}$ diverges to $+\infty$. Find examples of sequences $\{a_n\}$ and $\{b_n\}$ so that $\left\{\frac{a_n}{b_n}\right\}$
   (a) diverges to $+\infty$.
   (b) converges to 7.
   (c) converges to 0.
   (d) diverges to $-\infty$.
   (e) oscillates.