Problem Set 8 **Real Analysis 1** **Due 11/18/16**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Suppose that a function $f : [a, b] \to \mathbb{R}$ is bounded and $\mathscr{P} = x_0, x_1, x_2, ..., x_n$ is a partition of $[a, b]$. Then there exist $m, M \in \mathbb{R}$ such that

$$m(b - a) \leq L(\mathscr{P}, f) \leq S(\mathscr{P}, f) \leq U(\mathscr{P}, f) \leq M(b - a)$$

2. Suppose that a function $f : [a, b] \to \mathbb{R}$ is bounded and $\mathscr{P}$ and $\mathscr{Q}$ are partitions of $[a, b]$. Show that if $\mathscr{P} \subseteq \mathscr{Q}$, then $U(\mathscr{Q}, f) \leq U(\mathscr{P}, f)$.

3. Let $f(x) = x$, and let $\mathscr{P}$ be a regular partition with n subdivisions. Evaluate $U(\mathscr{P}, f)$ and $L(\mathscr{P}, f)$, and find their limits as n approaches ∞.

4. [6.1.3] Prove that a constant function $f(x) = c, c \in \mathbb{R}$, is Riemann integrable on any interval $[a, b]$ and $\int_a^b f(x)\, dx = c(b - a)$.

5. [6.1.4] If $f(x) \leq g(x) \leq h(x)$ for all $x \in [a, b]$, and f and h are Riemann integrable on $[a, b]$, then so is g.

6. [6.1.6] If a function $f : [a, b] \to \mathbb{R}$ is bounded and nonnegative, then $\underline{\int_a^b} f \geq 0$.