Exam 1 Calc 3 9/29/2017

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to y.

2. Suppose that w is a function of x, y, and z, each of which is a function of s and t. Write the Chain Rule formula for $\frac{\partial w}{\partial t}$. Make very clear which derivatives are partials.

3. Find an equation for the plane tangent to $f(x, y) = \sqrt{xy}$ at the point (1, 1, 1).

4. Show that $\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$ does not exist.

5. Let $f(x, y) = \dfrac{x}{x^2 + y^2}$. Find the maximum rate of change of f at the point (1,2) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, this Calc 3 stuff is killing me. There was this question on our practice exam about the direction derivatives, right? So like, if you know which direction is the level curve, and you know which direction is the gradient, and you face halfway in between them, then how big is the slope that way, right? So I obviously picked answer C, which said half as big as the gradient. But they said it was wrong, which is obviously because they're stupid, right?"

   Explain clearly to Biff what can be said about the directional derivative in the direction halfway between the direction of a level curve and the direction of the gradient, and why.

8. Find the maximum and minimum values of $f(x, y) = 2x^2 + 3y^2 - 2x + 5$ subject to the constraint $x^2 + y^2 = 4$.

9. Find the maximum and minimum values of $f(x, y) = 2x^2 + 3y^2 - 2x + 5$ subject to the constraint $x^2 + y^2 \leq 4$.

10. Find all critical points of $p(x, y) = 36xy\, e^{-2x - 3y}$ and classify them as maxima, minima, or saddle points.

Extra Credit (5 points possible):
For some values of a, the function $f(x, y) = 2x^2 + 3y^2 - a\,x + 5$ has two maxima subject to the constraint $x^2 + y^2 \leq 4$. For other values of a there is only a single maximum. Which are which?