Fake Exam 4 Calc 1 11/14/2018

Each problem is worth 0 points. For full credit learn enough to do well on the real exam.

1. Find the maximum value of $f(x) = 5x - x^2$.

   25/4 (which happens when $x = 5/2$)

2. Find the maximum value of $f(x) = 5x - x^2$ on [0,10].

   –50 (which happens when $x = 10$)

3. Find the largest interval on which $f(x) = x\,e^{-x}$ is increasing.

   $(-\infty, 1]$

4. Find the largest interval on which $f(x) = x\,e^{-x}$ is concave up.

   $[2, \infty)$

5. Find **all** local maxima of $f(x) = \sin x + \cos x$.

   $\pi/4 + \pi\, n$ for any integer n

6. Find the largest interval on which $f(x) = 3x^3 - 2x^2 + x - 7$ is decreasing.

   It's never decreasing, since the derivative is always positive.

7. Use Newton's method to calculate x_1, and x_2 for $\sqrt[3]{15}$ with $x_0 = 2$ as the initial approximation.

$x_1 = 31/12$
$x_2 = 42751/17298$ (aren't you glad you don't live back when there were no calculators?)

8. Squares with sides of length x are cut out of each corner of a rectangular sheet of metal measuring 6 ft by 4 ft. The resulting piece of cardboard is then folded into a box without a lid. The box is to be used as a container for mutant super piranhas, so it's only being filled to a level 6 inches below the brim to prevent escape. Find the largest volume of water that can be contained in such a box.

$x \approx 1.1068$, so $V \approx 4.1044$

9. Find the intervals on which the graph of $y = e^{-x^2/2}$ is concave up.

The intervals are $(-\infty, -1]$ and $[1, \infty)$.

10. Economists use terminology closely related to ours, so if $R(x)$ is a function that tells the revenue generated by selling x units, then they call the function $R'(x)$ the **marginal revenue function.** Similarly if $C(x)$ is a function that tells the cost to the company of producing x units, then they call the function $C'(x)$ the **marginal cost function.** They also get really excited about $P(x) = R(x) - C(x)$, the **profit** generated when selling x units.

    Show that profit is maximized when marginal cost is equal to marginal revenue.

Take the derivative, set it equal to zero, and stare at it until it's clear. (You have to assume something reasonable like that revenue per unit decreases as quantity increases to be totally proper about things, but that's not the real point here.)