Exam 1 **Calculus 3** **9/28/18**

Each problem is worth 10 points. Full credit requires good justification for your answers.

1. State the definition of the partial derivative of $f(x, y)$ with respect to y.

2. Suppose that f is a function of x, y, and z, each of which is a function of t. Write the Chain Rule formula for $\frac{df}{dt}$, and make very clear which derivatives are partials.

3. Write an equation for the plane tangent to $f(x, y) = x^2 + 3xy + 2y^2 + 6y - 4$ at the point $(1, -2)$.

4. Show that $\lim_{(x,y)\to(0,0)} \dfrac{3xy}{2x^2 + y^2}$ does not exist.

5. (a) Find the directional derivative of the function $g(p, q) = p^4 - p^2q^3$ at the point $(1, 2)$ in the direction of the vector $\vec{v} = \vec{i} - 3\vec{j}$.

(b) In which direction is the directional derivative greatest at the point (1,2)?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, Calc is so impossible! Even stuff that I thought made sense and was okay gets so confusing for what you actually need to be able to do, you know? Like, our professor was saying we wouldn't have to be able to draw the level curve diagrams, right? Because it's so hard to do it good by hand, right? But then he said it would be fair if it was one like for a plane, and now I'm totally panicing, because I have no idea what that would be like. Should I just drop and change my major?"

   Help Bunny out by explaining clearly what can be said about the level curves of a plane.

8. Find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $x + 2y = 6$.

9. Find all critical points of $f(x, y) = x^3 - 6xy + 8y^3$ and classify them as maxima, minima, or saddle points.

10. Find an equation for the plane tangent to the surface $x^2 + y^2 + xz + z^2 = 4$ at the point (1,1,1).

Extra Credit [5 points possible]: Find the highest point on the surface $x^2 + y^2 + xz + z^2 = 4$.