Exam 1 **Real Analysis 1** **9/26/18**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of a function $f(x)$ converging as x approaches a.

2. a) State the definition of an accumulation point.

b) Give an example of a set which has exactly one accumulation point.

3. a) State the definition of a Cauchy sequence.

b) Give an example of a sequence which is not Cauchy.

4. a) State the definition of an increasing function.

b) Give an example of a function which increases and converges as x approaches ∞.

5. A *field*, F is a nonempty set together with the operations of addition and multiplication, denoted by $+$ and $\cdot$, respectively, that satisfies the following eight axioms:

   (A1) (*Closure*) For all $a, b \in F$, we have $a + b, a \cdot b \in F$.

   (A2) (*Commutative*) For all $a, b \in F$, we have $a + b = b + a$ and $a \cdot b = b \cdot a$.

   (A3) (*Associative*) For all $a, b \in F$, we have $(a+b)+c = a+(b+c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

   (A4) (*Additive Identity*) There exists a zero element in F, denoted by 0, such that $a + 0 = a$ for any $a \in F$.

   (A5) (*Additive Inverse*) For each $a \in F$, there exists an element $-a$ in F, such that $a + (-a) = 0$.

   (A6) (*Multiplicative Identity*) There exists an element in F, which we denote by 1, such that $a \cdot 1 = a$ for any $a \in F$.

   (A7) (*Multiplicative Inverse*) For each $a \in F$ with $a \neq 0$ there exists an element in F denoted by $\frac{1}{a}$ or a^{-1} such that $a \cdot a^{-1} = 1$.

   (A8) (*Distributive*) For all $a, b, c \in F$, we have $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

   Prove, making explicit any of these axioms you use, that that multiplicative identity element in any field F is unique.

6. Show that any sequence which converges is bounded.

7. State and prove the Bolzano-Weierstrass Theorem for Sets.

8. Prove directly from the definition that

$$\lim_{x \to 5} x^2 = 25$$

9. Prove or give a counterexample: If $\{a_n\}$ is a Cauchy sequence and $S = \{a_n | n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is constant from some point on.

10. Definition: We say s_0 is a *bicumulation point* of a set S iff for any $\epsilon > 0$, there exist $s, t \in S \ni 0 < |s - s_0| < \epsilon$ and $0 < |t - s_0| < \epsilon$.

(a) Give an example of a bicumulation point which is not an accumulation point, or show that one cannot exist.
(b) Give an example of an accumulation point which is not a bicumulation point, or show that one cannot exist.