Exam 1 Calc 1 9/13/2019

Each problem is worth 10 points. For full credit provide complete justification for your answers.

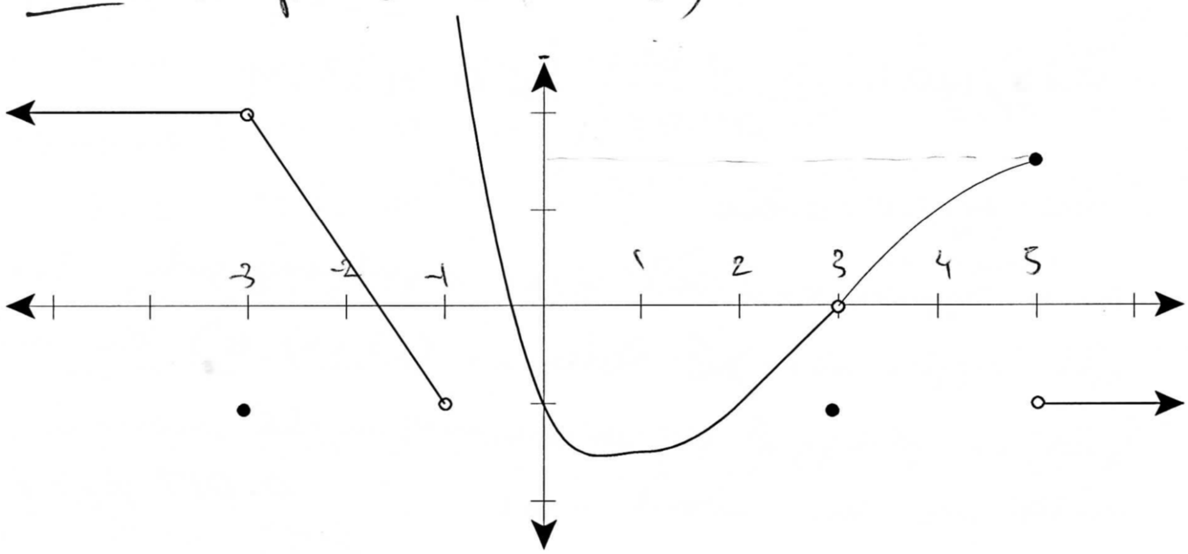
Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:
   a) $\lim_{x \to -3^-} g(x) = \underline{2}$
   b) $\lim_{x \to -3^+} g(x) = \underline{2}$
   c) $\lim_{x \to -3} g(x) = \underline{2}$ $\left(\lim_{x \to -3^-} g(x) = \lim_{x \to -3^+} g(x) \right)$
   d) $\lim_{x \to 5^+} g(x) = \underline{-1}$
   e) $\lim_{x \to 5^-} g(x) \approx \underline{1.5}$ Good
   f) $\lim_{x \to 5} g(x) = \underline{\text{DNE}}$

2. For which values of x does the function fail to be continuous?

$\underline{x = -3}$ (Removable discontinuous)
$\underline{x = -1}$ (infinite discontinuous)
$\underline{x = 3}$ (Removable discontinuous)
$\underline{x = 5}$ (Jump discontinuous)

Great



3. Evaluate $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$.

$= \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}}$

$= \lim_{x \to 5} (x+5)$

$= \lim_{x \to 5} x + \lim_{x \to 5} 5$ Sum law

$= 5 + 5$ Law x & Constant law

$= 10$

Excellent!

4. Let $f(x) = \frac{\sin x}{x}$. ***Make sure your calculator is in radian mode.*** Give answers accurate to at least 8 decimal places.

a) What is $f(0.1)$?

$\frac{\sin(0.1)}{0.1} \approx 0.99833417$

b) What is $f(0.01)$?

$\frac{\sin(0.01)}{0.01} \approx 0.99998333$

c) What is $f(0.001)$?

$\frac{\sin(0.001)}{0.001} \approx 0.99999983$

d) What is $\lim_{x \to 0^+} f(x)$?

As f(x) gets closer to zero,
it appears that the
points keep getting closer to 1

Great

5. An aid organization is building wells in remote areas, and then maintaining them, in order to improve the standard of living for people in the region. A well initially costs $10,000 for materials, and then on average $500 each year for maintenance after the first year. The company plans to have $400,000 per year to devote to the project. Let $w(t)$ express the number of wells that can be provided altogether after t years.

a) What is $w(1)$?

$$w(1) = \frac{400{,}000}{10{,}000} = 40 \text{ wells}$$

total

b) What is $w(2)$?

$w(2) = 78$ total number of wells

$$\frac{400{,}000 - 40(500)}{10{,}000} = 38 \text{ wells}$$

+ 40 wells

78

c) What is $\lim_{t\to\infty} w(t)$?

$$= \frac{400{,}000}{500} = 800 \text{ wells}$$

With out a roll over a budget of 400,000 will be able to maintain 800 wells, so $\lim_{t\to\infty} w(t) = 800$ wells

Good.

6. Evaluate $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$. $= \lim_{x\to\infty} \dfrac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}$

$$= \lim_{x\to\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x}\cdot\frac{1/x}{1/x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\cancel{1/x}}+3}$$

Excellent!

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Calc is totally killing me. I thought it would be easy because of multiple choice, right? But it's like they're all trick questions. There was this one, like, for how many different inputs closer and closer to something do you have to get outputs closer and closer to something for you to know that's what the limit is, right? So I said 3 because that's how many they used in the online homework, so that's pretty simple, right? But they said it's none of the above, which is pretty much crap, because it's gotta be *something*, right?"

Help Biff by explaining as clearly as you can the answer to his question.

There answer is none of the above because you can truly never be "close enough" for you to know with certainty. You would have to be infinitely close, because you can always be closer. Also, you have to be careful of what inputs you choose, because for example in many trig functions, just picking numbers "closer" does not give you an accurate representation of what is happening.

Good

8. Evaluate $\lim_{h \to 0} \frac{(5+h)^2 - 5^2}{h}$.

$$\lim_{h \to 0} \frac{5^2 + 10h + h^2 - 5^2}{h}$$

$$\lim_{h \to 0} \frac{10h + h^2}{h}$$

$$\lim_{h \to 0} 10 + h$$

$$10 + 0 = \boxed{10}$$

Good

9. A capacitor is charged to power a phone camera's flash, with the electric charge given by $Q(t) = Q_0\left(1-e^{-t/a}\right)$, where Q_0 and a are positive constants determined by the design of the phone. Evaluate $\lim_{t\to\infty} Q(t)$. Be sure to provide good justification for your conclusion.

$$\lim_{t\to\infty} Q_0\left(1-e^{-t/a}\right)$$

$\frac{-t}{a} \to -\infty$ as t is increasing and a is constant

$e^{-t/a} \to 0$ as taking e to a larger negative exponent gets you answers closer and closer to 0

Left with

$Q_0(1-0)$

Great

$$\lim_{t\to\infty} Q_0\left(1-e^{-t/a}\right) = \boxed{Q_0}$$

10. A car is driving down the highway at night when suddenly a tyrannosaurus appears in the middle of the road ahead. The driver immediately hits the brakes, and the distance between the car and dinosaur is given by $d(t) = 7.5t^2 - 90t + 240$ for values of t between 0 and 4.

Give answers accurate to at least 8 decimal places.

*assuming velocity is measured in m/s

a) Find the average velocity of the car over the interval [3.5, 4].

$$\frac{(7.5(4)^2 - 90(4) + 240) - (7.5(3.5)^2 - 90(3.5) + 240)}{0.5} = \boxed{-33.75 \text{ m/s}}$$

b) Find the average velocity of the car over the interval [3.9, 4].

$$\frac{(7.5(4)^2 - 90(4) + 240) - (7.5(3.9)^2 - 90(3.9) + 240)}{0.1} = \boxed{-30.75 \text{ m/s}}$$

c) Find the average velocity of the car over the interval [3.99, 4].

$$\frac{(7.5(4)^2 - 90(4) + 240) - (7.5(3.99)^2 - 90(3.99) + 240)}{0.01} = \boxed{-30.075 \text{ m/s}}$$

d) Estimate the car's instantaneous velocity at $t = 4$.

it seems that the car's velocity is approaching

-30 m/s at $t=4$

Excellent, except ft/s was intended, but yours is more fun - seeing a dinosaur ahead at 130 miles per hour is great. ☺