

Exam 4 Calc 1 11/15/2019

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the most general antiderivative of $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$.

$$f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$$

$$F(x) = \frac{1}{2}x + \frac{3}{4 \times 3}x^3 - \frac{4}{5 \times 4}x^4 + C$$

$$= \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$$

Good

2. Find the second derivative of $s(x) = \frac{e^x - e^{-x}}{2}$.

$$\frac{e^x - e^{-x}}{2}$$

First derivative of sinh = cosh
derivative of cosh = sinh

Excellent

3. Find the maximum and minimum values of $f(x) = 2x^3 - 15x^2 + 24x + 1$ on $[0, 3]$.

$$f(x) = 2x^3 - 15x^2 + 24x + 1 \quad [0, 3]$$

* Find critical points

$$f'(x) = 6x^2 - 30x + 24$$

$$= x^2 - 5x + 4$$

$$= (x-4)(x-1)$$

$$x = 1$$

$x = 4 \rightarrow$ not within interval

$$f(0) = 2(0)^3 - 15(0)^2 + 24(0) + 1 = 1$$

$$f(1) = 2(1)^3 - 15(1)^2 + 24(1) + 1 = 12$$

$$f(3) = 2(3)^3 - 15(3)^2 + 24(3) + 1 = -8$$

abs. min at $(3, -8)$ Great!
abs. max at $(1, 12)$

4. Use Newton's Method with the function $f(x) = x^2 - 2$ and initial value $x_0 = 2$ to calculate x_1 .

$$f(x) = x^2 - 2$$

$$f'(x) = 2x$$

$$x_0 = 2$$

Using Newton's Method:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{2^2 - 2}{2 \cdot 2}$$

$$= 2 - \frac{4 - 2}{4} = \boxed{\frac{3}{2}}$$

Great

5. The manager of a large apartment complex knows from experience that 80 units will be occupied if the rent is 450 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 5 dollar increase in rent. Similarly, one additional unit will be occupied for each 5 dollar decrease in rent. What rent should the manager charge to maximize revenue?

$$R = (80 + x)(450 - 5x)$$

$$= 36,000 - 400x + 450x - 5x^2$$

$$= -5x^2 + 50x + 36,000$$

~take derivative

$$R' = -10x + 50$$

~set equal to zero

$$0 = -10x + 50$$

$$10x = 50$$

$x = 5 \rightarrow$ this means that at max revenue, 5 more units will be occupied (85 total)

$$R = (80 + 5)(450 - 5(5))$$

$$R_{\max} = \$36,125$$

$$\text{rent} = \frac{36125}{85} = \boxed{\$425 \text{ per unit}}$$

6. For which values of t is $p(t) = \frac{e^t}{1+e^t}$ concave down?

$$p'(t) = \frac{e^t(1+e^t) - e^t(e^t)}{(1+e^t)^2}$$

$$\rightarrow p'(t) = \frac{e^t + e^{2t} - e^{2t}}{(1+e^t)^2} = \frac{e^t}{(1+e^t)^2}$$

$$\rightarrow p''(t) = \frac{e^t(1+e^t)^2 - 2(1+e^t)e^{2t}}{(1+e^t)^4}$$

$$p''(t) = \frac{(1+e^t)(e^t(1+e^t) - 2e^{2t})}{(1+e^t)^4}$$

$$p''(t) = \frac{e^t(1+e^t - 2e^t)}{(1+e^t)^3}$$

$$p''(t) = \frac{e^t(1-e^t)}{(1+e^t)^3}$$

$p(t)$ concaves down

$$\rightarrow p''(t) < 0$$

$$\rightarrow \frac{e^t(1-e^t)}{(1+e^t)^3} < 0$$

We have: $(1+e^t)^3 > 0 \forall t$

$$\rightarrow e^t(1-e^t) < 0$$

$$e^t > 0 \rightarrow (1-e^t) < 0$$

$$\rightarrow 1 < e^t$$

$$\rightarrow t > \ln 1 \rightarrow \boxed{t > 0}$$

Good.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG, this calculus stuff is soooo confusing! I was pretty okay with, you know, like the take a derivative and put it equal zero stuff, you know? But there was this question on the true/false part of our exam where, like, it said if f has an absolute minimum value at c , then $f'(c) = 0$, so I said true because that's totally how you do it, you know? But they said it's wrong and I think everything is totally unfair, you know?"

Explain clearly to Bunny what's going on with this true/false question.

Alright so you are partially right for assuming that in order for it to be a minimum or maximum its derivative must be zero because that usually is how it is understood. However, the value c as a minimum could be an endpoint if you're are looking in a given interval. Also, important to remember that critical points not only can be found when the derivative is zero, but when the derivative is undefined as well.

Great

8. Use Newton's Method with a first approximation of $x_0 = 1$ to generate approximations x_1 and x_2 for the value of $\ln 3$.

$$\ln(3) = x$$

$$e^{\ln(3)} = e^x$$

$$3 = e^x$$

$$0 = e^x - 3$$

$$f(x) = e^x - 3$$

$$f'(x) = e^x$$

$$x_1 = 1 - \frac{e^1 - 3}{e^1}$$

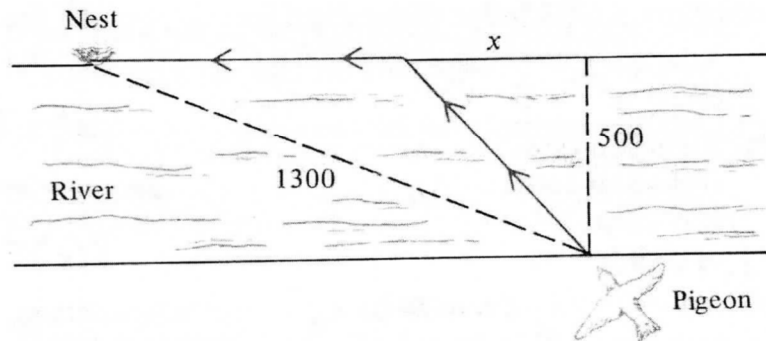
$$x_1 = 1.103638324$$

$$x_2 = 1.103638324 - \frac{e^{1.103638324} - 3}{e^{1.103638324}}$$

$$x_2 = 1.098624898$$

Great

9. [E&G] It is known that homing pigeons fly faster over land than over water. Assume that they fly 10 meters per second over land but only 8 meters per second over water. If a pigeon is located at the edge of a straight river 500 meters wide and must fly to its nest, located 1300 meters away on the opposite side of the river (shown below), what path would minimize its flying time?



$$T(x) = \frac{\sqrt{500^2 + x^2}}{8} + \frac{(1300 - x)}{10}$$

$$T'(x) = \frac{1}{8} \cdot \frac{1}{2} (250000 + x^2)^{-1/2} \cdot 2x + -\frac{1}{10}$$

$$0 = \frac{x}{8\sqrt{250000 + x^2}} - \frac{1}{10}$$

$$\frac{1}{10} = \frac{x}{8\sqrt{250000 + x^2}}$$

$$\frac{1}{100} = \frac{x^2}{64(250000 + x^2)}$$

$$16000000 + 64x^2 = 100x^2$$

$$16000000 = 36x^2$$

$$\frac{4000000}{9} = x^2$$

$$x = \frac{2000}{3}$$

$$x \approx 666.\bar{6}$$

10. Jon is consulting as a forensic mathematician to supplement his meager income. In one case, a body has been thrown out a window of a high-rise building, and Jon needs to determine which floor it came from. A knife apparently fell out the same window, and stuck into the asphalt to a depth suggesting it hit the ground with a velocity of 96 feet per second. From what height was it dropped?

$$a = h''(t) = \underline{-32} \text{ ft/s}^2$$

$$v = h'(t) = \underline{-32t} + \underline{C} \text{ ft/s}$$

$$h = h(t) = \underline{-16t^2} + \underline{Ct} + \underline{D} \text{ ft}$$

When $h(t) = 0 \rightarrow h'(t) = -96 \text{ ft/s}$

$$\rightarrow -16t^2 + Ct + D = 0$$

$$h'(t) = -96 \rightarrow -32t + C = -96$$

$$\underline{-32t + 96 = C = -96 + 32t}$$

$$h(t) = -16t^2 + (-96 + 32t)t + D = 0$$

$$= -16t^2 - 96t + 32t^2 + D = 0$$

$$= 16t^2 - 96t + D = 0 \quad (1)$$

Correct

We have at $t=0 \rightarrow v=0 \rightarrow h'(t)=0$

$$\rightarrow -32 \cdot 0 + C = 0 \rightarrow C = 0$$

$$\rightarrow h'(t) = -96 \rightarrow -32t = -96 \rightarrow t = 3$$

We have (1) $\rightarrow 16t^2 - 96t + D = 0$

$$16(3)^2 - 96(3) = -D \rightarrow$$

$$\boxed{D = 144 \text{ ft}}$$