Exam 1 Calc 3 9/27/2019

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to y.

2. Suppose that w is a function of x and y, each of which is a function of s, t, u, and v. Write the Chain Rule formula for $\dfrac{\partial w}{\partial v}$. Make very clear which derivatives are partials.

3. Find the directional derivative of $f(x, y) = \sqrt{xy} + \dfrac{x}{y}$ at the point (12,3) in the direction of $\mathbf{v} = \langle -3,4 \rangle$.

4. Show that $\lim\limits_{(x,y)\to(0,0)} \dfrac{(-5x+y)^2}{25x^2+y^2}$ does not exist.

5. Let $f(x, y) = \dfrac{x}{x^2 + y^2}$. Find the maximum rate of change of f at the point (2,3) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, this Calc 3 stuff is killing me. There was this question on our practice exam about, like, how many different surfaces could have the same level curves? And I figured it was a trick question so I said yes lots, but I have no clue why that's right. I mean, is it like, there's exactly three surfaces with the same level curves, or more, or what?"

   Explain clearly to Biff whether there is only one function with a particular set of level curves, or if there can be two or more.

8. Find and classify all critical points of $f(x, y) = 3x^2 - y^3 - 6xy + 5$.

9. Find the extreme values of $f(x, y) = x^2 + y^2 - 2x + 4y - 1$ subject to the constraint $x^2 + y^2 \leq 9$.

10. At what point(s) on the surface

$$y = x^2 + z^2$$

is the tangent plane parallel to the plane

$$x + 2y + 3z = 1?$$

Extra Credit (5 points possible):

What's going on with the directional derivatives of $f\left(x, y\right) = \sqrt[3]{xy}$ at (0,0)?