

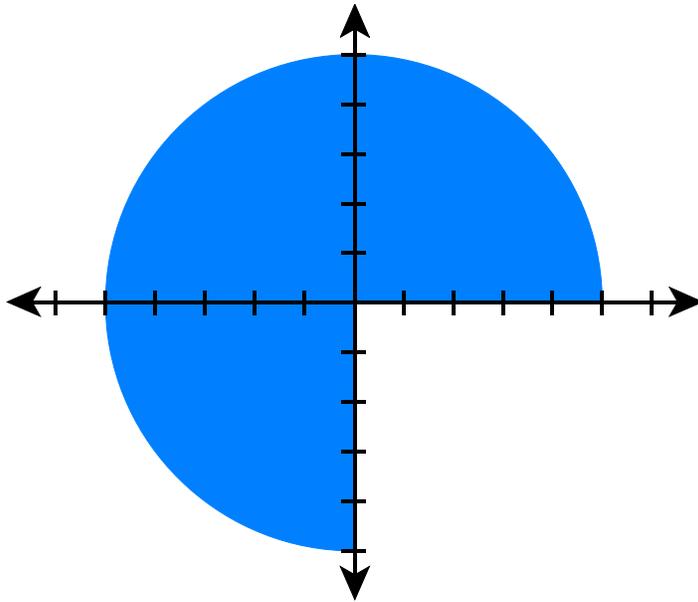
**Exam 2    Calc 3    10/25/2019**

Each problem is worth 10 points. For full credit provide complete justification for your answers. All integrals should be set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no  $x$  or  $y$ , etc.

1. Write a double Riemann sum for  $\iint_R f \, dA$ , where  $R = \{(x, y) : 0 \leq x \leq 4, 2 \leq y \leq 6\}$  using midpoints with  $n = m = 2$  subdivisions.

2. Let  $f(x, y) = 4x^2 + 9y^2$ . Let  $R$  be the parallelogram with vertices  $(0, 0)$ ,  $(2, 2)$ ,  $(2, 5)$ , and  $(0, 3)$ . Set up an iterated integral for  $\iint_R f \, dA$ .

3. Set up an iterated integral for the total mass of a plate shaped like the region shown below, with density  $\rho(x, y) = k$ .



4. Set up limits of integration for an iterated integral for the volume of the region above the  $xy$ -plane, outside a sphere of radius 1 centered at the origin, and inside a sphere of radius 2 centered at the origin.

5. Evaluate  $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 8 \, dy \, dx$ .

6. Evaluate  $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1} dy dx$ .

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Calc 3 is just so much! It's like, these things I thought I understood before are more confusing now, like with polar? So like sometimes you go zero to  $2\pi$ , but sometimes you go 0 to  $\pi$  over two and take it times four, but sometimes you can't do that. How do you know which?"

Explain clearly to Bunny an example where her two ways are equal, and also an example where they are not, and why.

8. Set up iterated integrals for the centroid of the solid bounded by  $z = 9 - \sqrt{x^2 + y^2}$  above the plane  $z = 0$  and below the plane  $z = b$  for some value  $0 \leq b \leq 9$ .

9. Find the Jacobian for the transformation from rectangular to spherical coordinates.

10. Set up one or more iterated integrals for the volume of the region inside both the cylinder  $x^2 + z^2 = 1$  and  $y^2 + z^2 = 1$ .

Extra Credit (5 points possible):

Why are  $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^3 \sin^2 \phi \cos \theta d\rho d\phi d\theta$  and  $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^3 \sin \phi \cos \phi d\rho d\phi d\theta$  equal?