Examlet 2 **Foundations of Advanced Math** **10/4/19**

1. Write each of the sets below as simply as possible:

   (a) What is $\{1,3\} - \{2,4\}$?

   $A \wedge B'$

   $\underline{\{1,3\}}$

   (b) What is $(1,3) - (2,4)$?

   $\underline{(1,2]}$

   (c) What is $[1,5] \cap [5,8]$?

   $\underline{\{5\}}$

   (d) What is $\{1,2\} \times \{2,4\}$?

   $\underline{\{(1,2),(2,2),(1,4),(2,4)\}}$

   (e) What is $\mathcal{P}\{1,2\}$?

   $\underline{\{\emptyset,\{1\},\{2\},\{1,2\}\}}$

   Excellent!

2.

$$\left(\bigcup_{i\in I} A_i\right)' = \bigcap_{i\in I} A_i'$$

Well, take $x \in \left(\bigcup_{i\in I} A_i\right)'$, so $x \in \mathbb{X}$ and $\neg\left(x \in \bigcup_{i\in I} A_i\right)$,

so $\neg\left(\exists i \in I, x \in A_i\right)$

so $\forall i \in I, \neg x \in A_i$ by the box on p. 13

so $\forall i \in I, x \in A_i'$

so $x \in \bigcap_{i\in I} A_i'$

Therefore $\left(\bigcup_{i\in I} A_i\right)' \subseteq \bigcap_{i\in I} A_i'$

Now take $x \in \bigcap_{i\in I} A_i'$, so $\forall i \in I, x \in \mathbb{X}$ and $x \in A_i'$

so $\forall i \in I, \neg x \in A_i$

so $\neg \exists i \in I, x \in A_i$ by the box on p. 13

so $\neg\left(x \in \bigcup_{i\in I} A_i\right)$

so $x \in \left(\bigcup_{i\in I} A_i\right)'$

Therefore $\bigcap_{i\in I} A_i' \subseteq \left(\bigcup_{i\in I} A_i\right)'$, and thus $\left(\bigcup_{i\in I} A_i\right)' = \bigcap_{i\in I} A_i'$. $\square$

3. (a) $\forall a, b, c, d \in \mathbb{R}, a > b \wedge c > d \Rightarrow a + c > b + d.$

$a > b$

$a+c > b+c$

$c > d$

$b+c > b+d$

CAP

$a+c > b+c > b+d$

$a+c > b+d$. $\square$ TPI

Great

(b) $\forall a, b, c, d \in \mathbb{R}, a > b \wedge c > d \Rightarrow a - c > b - d.$

Counter Example:

$a > b$

$a=4 \quad b=0$

$c > d$

$c=-2 \quad d=-7$

$a-c \overset{?}{>} b-d$.

$4--2 \overset{?}{>} 0--7$

$4+2 \overset{?}{>} 0+7$

$6 \not> 7$. $\square$

4. $\forall i \in \mathbb{Z}^+$ let $A_i = [0, 1/n]$

(a) What is $\bigcap_{i\in\{1,2,3\}} A_i$?

$[0, \frac{1}{1}] \cap [0, \frac{1}{2}] \cap [0, \frac{1}{3}]$

$\underline{[0, \frac{1}{3}]}$ Good

(b) What is $\bigcap_{i\in\mathbb{Z}^+} A_i$?

$\bigcap_{i\in\mathbb{Z}^+} A_i$

$\{x \mid x \in A_i \text{ for all } i\}$

As i increases, $1/n$ will be closer and closer to 0, but will never reach it.

$\bigcap_{i\in\mathbb{Z}^+} A_i = \underline{\{0\}}$ Great

5. $\forall x \in \mathbb{R}, -|x| \le x \le |x|$.

$$\begin{cases} \text{if } x \geq 0 & x \\ \text{if } x < 0 & -x \end{cases}$$

Say $x \geq 0$. Then $|x| = x$.

So $x \leq |x|$

$x \geq 0$

$|x| \geq 0$

$0 \geq -|x|$

$x \geq 0 \geq -|x|$

So $x \geq -|x|$

So $-|x| \leq x \leq |x|$ if $x \geq 0$.

Say $x < 0$. Then $|x| = -x$.

So $x = -|x|$

So $x \geq -|x|$

$x < 0$

$-|x| < 0$

$0 < |x|$

$x < 0 < |x|$

So $x < |x|$

So $x \leq |x|$

So $-|x| \leq x \leq |x|$ if $x < 0$.

So by Trichotomy, $\forall x \in \mathbb{R}, -|x| \leq x \leq |x|$ $\square$

Good.