Examlet 4 **Foundations of Advanced Math** **11/15/19**

1. Consider the relation $\sim$ on $\mathbb{R}$ defined by $x \sim y \Leftrightarrow |x - y| \leq 4$.

   (a) Find 3 elements of $\mathbb{R}$ that are related to 2.

   (b) Find 3 elements of $\mathbb{R}$ that are not related to 2.

   (c) Determine whether $\sim$ is an equivalence relation.

2. Let $S = \{a, b, c, d\}$, and let $\sim = \{(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)\}$.

   (a) Give the equivalence classes of $\sim$.

   (b) Give the partition associated with $\sim$.

3. Let S be a set and Π a partition of S. Let $\sim$ be a relation on S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$.

   (a) Show $\sim$ is a reflexive relation.

   (b) Show $\sim$ is a symmetric relation.

   (c) Show $\sim$ is a transitive relation.

4. (a) Give all (unlabeled) trees with $n \leq 5$ vertices.

(b) The number of edges in a tree with n vertices is $n - 1$.

5. Call a graph **quintic** iff every vertex in that graph has degree 5. Then the number of vertices in any quintic graph must be even.