Problem Set 10 **Foundations** **Due 12/6/19**

Five of these problems will be graded, with each problem worth 4 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. $\forall x, y \in N, x + (y + 0) = (x + y) + 0$
2. $\forall x, y, z \in N, x + (y + z) = (x + y) + z \Rightarrow x + (y + z') = (x + y) + z'$
3. $\forall x, y, z \in N, x + (y + z) = (x + y) + z$
4. $\forall y \in N, 0 + y = y + 0$
5. $\forall x, y \in N, x + y = y + x \Rightarrow x' + y = y + x'$
6. $\forall x, y \in N, x + y = y + x$
7. Using the definition of $S(A)$ from section 5.6, write $S(\emptyset), S(S(\emptyset)), S(S(S(\emptyset)))$, and $S(S(S(S(\emptyset))))$ explicitly. How many elements are in each of these sets?
8. With the understanding that $0' = 1, 1' = 2, 2' = 3$, and $3' = 4$, where these are elements of a Peano system, show that $2 + 2 = 4$.