Exam 1 **Modern Algebra 1** **9/16/20**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of the lettered problems you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. State the definition of a group.

2. Let $\alpha : S \to T$ be a mapping. State the definition of the inverse mapping of α.

3. Give an example of a subgroup of $\mathbb{Z}$ with addition that is not $\mathbb{Z}$ itself. Explain how you know it's a subgroup.

4. In S_3, let $\alpha = (1\ 3\ 2)$ and $\beta = (3\ 1)$.

   (a) Find α^{-1}.

   (b) Compute $\alpha \circ \beta$

5. Show that the identity element in a group is unique.

6. Let $\mathbb{Z}$ be the set of integers, and define $*$ on $\mathbb{Z}$ as: $m * n = m + n + mn$.

   (a) Is $*$ commutative?

   (b) Does $*$ have an identity?

7. Let G be a group with operation $*$, and let H be a subset of G. Show that H is a subgroup of G iff

   (a) H is nonempty,

   (b) if $a \in H$ and $b \in H$, then $a * b \in H$, and

   (c) if $a \in H$, then $a^{-1} \in H$.

□ A. Determine, with proof, whether the set of 2 by 2 matrices of the form

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

with a, b, c positive real numbers, forms a group with matrix multiplication.

☐ B. Let G be a group, and $a \in G$. Suppose for some (one) $b \in G$, $a * b = b$. Is it necessarily the case that $a = e$, the identity of G?

□ C. Let G be a group, and let $Z(G) = \{x \in G : x * a = a * x \text{ for all } a \in G\}$. Show $Z(G)$ is a subgroup of G.

□ D. Give three distinct subgroups of S_4.

□ E. Write (1 2 4 6 5) as a product of two-cycles.

□ F. Solve the equation $(1\ 2\ 3) \circ x \circ (4\ 1) = (1\ 5\ 2)$, that is, find an element x of S_5 that makes the equation true.