

Exam 3 Calc 1 10/21/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Let $f(x) = x^4 + e^x$ What is $f'(x)$?

2. a) What is $(2^x)'$?

b) What is $(\log_{10} x)'$?

3. What is the derivative of $\ln(\cos x)$?

4. a) If $f(x) = x^3 \arcsin x$, what is $f'(x)$?

b) If $g(x) = \arcsin(x^3)$, what is $g'(x)$?

5. Evaluate $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$.

6. Show why the derivative of $\arctan x$ is what it is.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is only for geniuses! At first I thought the elope-it-all rule thing was really easy, but on our exam I guess I really messed up. I did it like this, and the grader gave me zero. That's so unfair!"

$$\lim_{x \rightarrow (\frac{\pi}{2})^+} \frac{\cos x}{1 - \sin x} \stackrel{\text{"h}}{=} \lim_{x \rightarrow (\frac{\pi}{2})^+} \frac{-\sin x}{0 - \cos x} \stackrel{\text{"h}}{=} \lim_{x \rightarrow (\frac{\pi}{2})^+} \frac{-\cos x}{\sin x} = \frac{0}{1}$$

Explain clearly to Bunny what she should understand about using L'Hôpital's Rule here.

8. Evaluate $\lim_{x \rightarrow \infty} x^2 e^{-x}$. Provide good justification for your steps.

9. Strontium-90 is a biologically important radioactive isotope that is created in nuclear explosions. It has a half life of 28 years. How many years would it take to reduce the amount of strontium-90 created in a particular explosion to a thousandth of the initial amount?

10. a) The population of India was 439 million in 1961 and 548 million in 1971. Use an exponential model to predict the population of India in 1981.

b) When would you predict the population of India would reach 1 billion?

Extra Credit (5 points possible):

a) What's the derivative of $x \tan^{-1} x$?

b) What's the derivative of $\ln(1 + x^2)$?

c) Is there a function whose derivative is $\tan^{-1} x$?

