Exam 3   Calc 1   10/21/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Let $f(x) = x^4 + e^x$ What is $f'(x)$?

$f(x) = x^4 + e^x$

$f'(x) = 4x^3 + e^x$ e^x is its own dirivative and dirivative of x^4 via power rule nx^{n-1} is $4x^3$

Excellent

2. a) What is $(2^x)'$?

$(2^x)' = \ln 2 (2^x)$

Great

b) What is $(\log_{10} x)'$?

$(\log_{10} x)' = \frac{1}{x \cdot \log 10}$

3. What is the derivative of ln(cos x)?

$f(x) = \ln(\cos(x))$

Chain Rule $f'(x) = \frac{1}{\cos(x)} \cdot -\sin(x)$ Excellent

4. a) If $f(x) = x^3 \arcsin x$, what is $f'(x)$? $(\arcsin)' = \frac{1}{\sqrt{1-x^2}}$ product rule

$3x^2 \cdot \arcsin x + x^3 \cdot \frac{1}{\sqrt{1-x^2}}$

$f'(x) = 3x^2 \arcsin x + \frac{x^3}{\sqrt{1-x^2}}$ Excellent!

b) If $g(x) = \arcsin(x^3)$, what is $g'(x)$? chain rule

$g'(x) = \frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2$

5. Evaluate $\lim_{x\to 0^+} \frac{\ln x}{x}$.

from the right

$y = \lim_{x \to 0^+} \left(\frac{\ln x}{x}\right)$

x	y
1	0
0.5	~ -1.38629
0.1	~ -23.02585
0.01	~ -460.517
0.0001	~ -92103.4

$\lim_{x \to 0^+} \left(\frac{\ln x}{x}\right) = -\infty$

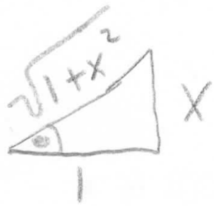
smaller positive x in ln(x) = greater negative output

6. Show why the derivative of arctan x is what it is.

I know, $\tan(\arctan x) = x$

differentiate $\sec^2(\arctan x)\cdot(\arctan x)' = 1$

$$(\arctan x)' = \frac{1}{\sec^2(\arctan x)}$$



$$(\arctan x)' = \frac{1}{1+x^2}$$

sec is hypotenuse over adjacent

$$\left(\frac{\sqrt{1+x^2}}{1}\right)^2$$

Excellent!

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is only for geniuses! At first I thought the elope-it-all rule thing was really easy, but on our exam I guess I really messed up. I did it like this, and the grader gave me zero. That's so unfair!"

$$\lim_{x\to(\frac{\pi}{2})^+} \frac{\cos x}{1-\sin x} \overset{l'h}{=} \lim_{x\to(\frac{\pi}{2})^+} \frac{-\sin x}{0-\cos x} \overset{l'h}{=} \lim_{x\to(\frac{\pi}{2})^+} \frac{-\cos x}{\sin x} = \frac{0}{1}$$

Explain clearly to Bunny what she should understand about using L'Hôpital's Rule here.

L'Hopital's rule can only be used when the numerator and denominator are in a race towards infinity or zero

$\left(\lim_{x\to\frac{\pi}{2}^+} \frac{\cos x}{1-\sin x}\right)$ works fine because of this.

However when Bunny gets to $\left(\lim_{x\to\frac{\pi}{2}^+} \frac{-\sin x}{0-\cos x}\right)$ this is not something we can apply L'Hopital's rule to.

Great

8. Evaluate $\lim_{x\to\infty} x^2e^{-x}$. Provide good justification for your steps.

$$\lim_{x\to\infty} x^2e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$$

Good

9. Strontium-90 is a biologically important radioactive isotope that is created in nuclear explosions. It has a half life of 28 years. How many years would it take to reduce the amount of strontium-90 created in a particular explosion to a thousandth of the initial amount?

$m(t) = Ae^{kt}$

$m(0) = 1000$
$m(28) = 500$

$m(0) = 1000 = 1000e^{k(0)}$

$m(28) = 500 = 1000e^{k(28)}$

$\ln 0.5 = 28k$

$k = \frac{\ln .5}{28}$

$1 = 1000e^{\frac{\ln .5}{28}(t)}$

$\frac{1}{1000} = e^{\frac{\ln 0.5}{28}(t)}$

$\ln \frac{1}{1000} = \frac{\ln 0.5}{28} \cdot t$

$t = (\ln \frac{1}{1000}) / \frac{\ln 0.5}{28} = 279.04196$

Excellent

$\boxed{279.04 \text{ years}}$

10. a) The population of India was 439 million in 1961 and 548 million in 1971. Use an exponential model to predict the population of India in 1981.

$P(t) = Ae^{kt}$

$P(0) = 439 = 439e^{kt}$

$P(10) = 548 = 439e^{k(10)}$

$\ln \frac{548}{439} = 10k$

$k = \left(\frac{\ln \frac{548}{439}}{10}\right) = 0.02218$

$P(20) = 439e^{0.02218(20)} = \boxed{684.097}$

Population in 1981 $\approx$ 684.097

b) When would you predict the population of India would reach 1 billion?

$1000 = 439e^{0.02218t}$

$\ln \frac{1000}{439} = .02218t$

Excellent

$t = \left(\frac{\ln \frac{1000}{439}}{.02218}\right) = \boxed{37.117}$

$1961 + 37 = \boxed{1998}$

India will reach a billion in 1998