Exam 4   Calc 1   Due by 1:15pm on 11/11/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers. You may consult any textbook, notes, or pre-existing internet sources, but may not consult with any other humans, directly or indirectly, while taking this exam. When you're done, upload a pdf of the entire exam to Moodle.

1. If $f(x) = 9x - x^2$, at what x- value does the maximum value of $f(x)$ occur?

2. Given the following information about a continuous function $g(x)$, determine the intervals of increase and decrease and intervals of concavity of $g(x)$:

	$(-\infty,-3)$	$(-3,0)$	$(0,2)$	$(2, +\infty)$
$g'(x)$	+	+	−	−
$g''(x)$	+	−	−	+

3. Let $f(x) = 3 - x^3 + 2x^2$. Find the absolute extrema of f on the interval $[0, 3]$.

4. Find all intervals where $f(x) = 3 - x^3 + 2x^2$ is decreasing.

5. A farmer wants to create a rectangular lot for his emu herd. He has 4800 feet of fence, and wants to have a dividing fence down the middle of the lot (parallel to one of the outside edges) to keep the males and females separated. What is the largest area that can be created?

6. Use Newton's Method with an initial approximation of $x_1 = 2$ to find x_2, the second approximation to a root of the function $f(x) = x^3 - x^2 - 2$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. I kinda like the Newton's Method thing, you know? I mean, there's definite answers to work out. But then the professor was saying something about times when Newton's Method fails to find a solution even though it's there, and it totally confused me. How could it fail, dude, it's a formula?"

Help Biff understand at least one situation where Newton's Method fails to find a root of a function, even though the function does have a root..

8. A discount theater has been charging $3 for tickets, and averaging 80 patrons per show. They expect from experience that if they increase ticket prices, for each $1 of price increase they'll sell 20 fewer tickets per show. What price should they set to maximize revenue from each show?

9. Let $f(x) = \dfrac{2e^x}{2e^x + 12}$. Find all inflection points of f, and describe the interval(s) where f is concave down.

10. a) Let $f(x) = x^3 - 3x^2 + 2x$. Find all inflection points of f.

b) Suppose that f is a third degree polynomial which crosses the x-axis at three distinct points a, b, and c. Show that f must have an inflection point at $(a + b + c)/3$.

Extra Credit (5 points possible):
a) Find a function with a tangent line that intersects the graph at a point where the graph is horizontal.

b) Explain why such a graph is important.