Exam 4 Calc 1 Due by 1:15pm on 11/11/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers. You may consult any textbook, notes, or pre-existing internet sources, but may not consult with any other humans, directly or indirectly, while taking this exam. When you're done, upload a pdf of the entire exam to Moodle.

1. If $f(x) = 9x - x^2$, at what x- value does the maximum value of $f(x)$ occur?

$f'(x) = 9 - 2x$

$0 = 9 - 2x$

$2x = 9$

$x = \frac{9}{2}$ or 4.5

$f''(x) = -2$ ∴ concave down, so the critical point is a max.

2. Given the following information about a continuous function $g(x)$, determine the intervals of increase and decrease and intervals of concavity of $g(x)$:

	$(-\infty,-3)$	$(-3,0)$	$(0,2)$	$(2,+\infty)$
$g'(x)$	+	+	–	–
$g''(x)$	+	–	–	+

Increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$

Concave up on $(-\infty, -3) \cup (2, \infty)$

Concave down on $(-3, 2)$

3. Let $f(x) = 3 - x^3 + 2x^2$. Find the absolute extrema of f on the interval $[0, 3]$.

$f'(x) = -3x^2 + 4x$

$0 = x(4 - 3x)$

$x = 0$ or $x = 4/3$

$f(0) = 3$

$f(3) = -6$

$f(4/3) = \frac{113}{27} \approx 4.18519$

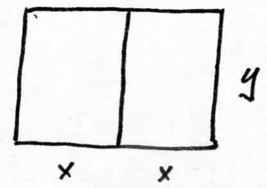
4. Find all intervals where $f(x) = 3 - x^3 + 2x^2$ is decreasing.

$0 \qquad 4/3$

$f' \qquad - \qquad + \qquad -$

Decreasing on $(-\infty, 0) \cup (4/3, \infty)$

5. A farmer wants to create a rectangular lot for his emu herd. He has 4800 feet of fence, and wants to have a dividing fence down the middle of the lot (parallel to one of the outside edges) to keep the males and females separated. What is the largest area that can be created?



$4x + 3y = 4800$

$3y = 4800 - 4x$

$y = 1600 - \frac{4}{3}x$ $\quad \cancel{2400 - 2x}$

Area $= 2x \cdot y$

$A(x) = 2x\left(1600 - \frac{4}{3}x\right)$

$A(x) = 3200x - \frac{8}{3}x^2$

$A'(x) = 3200 - \frac{16}{3}x$

$0 = 3200 - \frac{16}{3}x$

$\frac{16}{3}x = 3200$

$x = \frac{9600}{16} = 600$

$y = 1600 - \frac{4}{3}(600)$

$= 1600 - 800$

$= 800$

Area $= 2(600)(800)$

$= 960000 \text{ ft}^2$

6. Use Newton's Method with an initial approximation of $x_1 = 2$ to find x_2, the second approximation to a root of the function $f(x) = x^3 - x^2 - 2$.

$$f'(x) = 3x^2 - 2x$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = (2) - \frac{f(2)}{f'(2)}$$

$$x_2 = 2 - \frac{2}{8}$$

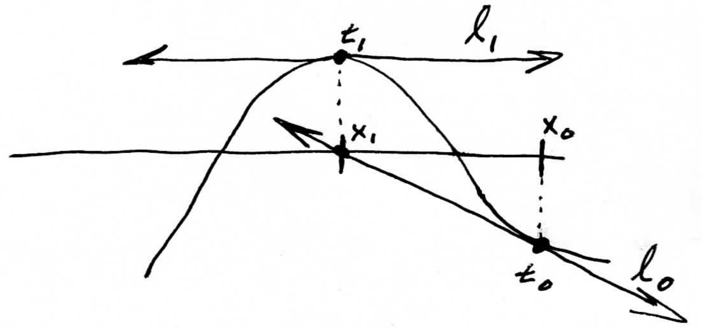
$$x_2 = 2 - \frac{1}{4}$$

$$x_2 = \frac{7}{4}$$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. I kinda like the Newton's Method thing, you know? I mean, there's definite answers to work out. But then the professor was saying something about times when Newton's Method fails to find a solution even though it's there, and it totally confused me. How could it fail, dude, it's a formula?"

Help Biff understand at least one situation where Newton's Method fails to find a root of a function, even though the function does have a root..

Biff, the thing is, even if it's a formula, it's an iterative thing, which means you repeat it over and over. That makes for a lot of places where things can go wrong. Imagine a graph like this one, okay?



So Newton's Method is supposed to find better and better approximations for where a function crosses the x-axis, right? But sometimes it gets tricked. If the starting guess is x_0, then Newton's uses the tangent line at t_0, which is l_0 in the picture, right? So it follows to where l_0 hits the axis, which is at x_1. Then you find the point on f with x-coordinate x_1, which is t_1, and you find the tangent line l_1. Now you're supposed to follow it to where it hits the x-axis and keep repeating... but it's parallel to the x-axis so it never hits it.

So that's a drastic case, but it shows how sometimes instead of improving, your next approximation can screw up and not be closer to where the graph hits the axis. It's rare, but it can happen.

8. A discount theater has been charging \$3 for tickets, and averaging 80 patrons per show. They expect from experience that if they increase ticket prices, for each \$1 of price increase they'll sell 20 fewer tickets per show. What price should they set to maximize revenue from each show?

$$R(x) = (3+x)(80-20x)$$

$$R(x) = 240 - 60x + 80x - 20x^2$$

$$R(x) = 240 + 20x - 20x^2$$

$$R'(x) = 20 - 40x$$

$$0 = 20 - 40x$$

$$x = \frac{1}{2}$$

So the optimal price would be \$3.50

9. Let $f(x) = \dfrac{2e^x}{2e^x + 12}$. Find all inflection points of f, and describe the interval(s) where f is concave down.

$$f'(x) = \frac{2e^x(2e^x+12) - 2e^x \cdot 2e^x}{(2e^x+12)^2}$$

$$f'(x) = \frac{4e^{2x} + 24e^x - 4e^{2x}}{(2e^x+12)^2}$$

$$f'(x) = \frac{24e^x}{(2e^x+12)^2}$$

$$f''(x) = \frac{24e^x(2e^x+12)^2 - 24e^x \cdot 2(2e^x+12) \cdot 2e^x}{(2e^x+12)^4}$$

$$f''(x) =$$

10. a) Let $f(x) = x^3 - 3x^2 + 2x$. Find all inflection points of f.

$$f'(x) = 3x^2 - 6x + 2$$
$$f''(x) = 6x - 6$$
$$0 = 6x - 6$$
$$x = 1$$

b) Suppose that f is a third degree polynomial which crosses the x-axis at three distinct points a, b, and c. Show that f must have an inflection point at $(a + b + c)/3$.

So a third degree polynomial that crosses the x-axis at a, b, and c would be like

$$f(x) = (x-a)(x-b)(x-c)$$

So the derivative is:

$$f'(x) = 1\cdot(x-b)(x-c) + (x-a)\cdot 1\cdot(x-c) + (x-a)\cdot(x-b)\cdot 1$$

And the second derivative is:

$$f''(x) = 1\cdot(x-c) + (x-b)\cdot 1 + 1\cdot(x-c) + (x-a)\cdot 1 + 1\cdot(x-b) + (x-a)\cdot 1$$

Or

$$f''(x) = 2(x-a) + 2(x-b) + 2(x-c)$$

An inflection point can only come where $f''(x) = 0$, so

$$0 = 2x - 2a + 2x - 2b + 2x - 2c$$
$$2a + 2b + 2c = 6x$$
$$x = \frac{a+b+c}{3}$$