Exam 1 Calc 3 9/25/2020

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Suppose that w is a function of x, y, and z, each of which is a function of s and t. Write the Chain Rule formula for $\dfrac{\partial w}{\partial t}$. Make very clear which derivatives are partials.

3. The function f has continuous second derivatives, and a critical point at (3, 6). Suppose $f_{xx}(3,6) = -4$, $f_{xy}(3,6) = -6$, $f_{yy}(3,6) = -9$. Classify the critical point at (3, 6).

4. Show that $\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$ does not exist.

5. Let $f(x, y) = \dfrac{x}{x^2 + y^2}$. Find the maximum rate of change of f at the point (1,2) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, this Calc 3 stuff is killing me. There was this question on our practice exam about the level curves and the maxes and stuff, right? And it said if there's this level curve picture, right, and the outside curve is the lowest, then the critical point in the middle has to be a max, right? Which is totally right, 'cause that would be like a parabala thing, with the peak up on top, right?"

   Explain clearly to Biff what can be said about the critical point in this situation.

8. a) Find an equation for the plane tangent to the paraboloid $z = x^2 + y^2$ at the point $(-1, 2)$.

b) Find all points on the surface $z = x^2 - y^2$ where the tangent plane is parallel to the one from part a.

9. Let $T(x, y) = -15\cos(\pi\, x/12 - \pi/4) + y/50 + 80$. Find the directional derivative of T in the direction of the vector $\langle 1,60 \rangle$ at the point $(10,0)$.

10. Find all critical points of $f(x,y) = xy(1 - x - y)$ and classify them as maxima, minima, or saddle points.

Extra Credit (5 points possible):

Is there a path of approach to the origin along which $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$ approaches -1?