Exam 3 Calculus 3 11/20/2020

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible. Upload your completed work as a pdf on Moodle. Don't panic.

1. Parametrize and give bounds for a path C which traverses the third quadrant portion of a circle (centered at the origin) counterclockwise from $(-5, 0)$ to $(0, -5)$.

2. Let $\mathbf{F}$ be the vector field $\mathbf{F} = 2xy\,\mathbf{i} + x^2\,\mathbf{j}$. Let C be the line segment from (5, 3) to (5, 7). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Let **F** be the vector field $\mathbf{F} = 2xy\,\mathbf{i} + 0\,\mathbf{j}$. Let C be the line segment from (5, 3) to (5, 7). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

4. Let C be the counterclockwise circle $x^2 + y^2 = 4$. Evaluate the line integral
$\oint_C \langle 12x, 5y \rangle \cdot d\mathbf{r}$.

5. Let C be the counterclockwise circle $x^2 + y^2 = 4$. Evaluate the line integral

$$\oint_C \langle 12y, 5x \rangle \cdot d\mathbf{r}\ .$$

6. If you have 5 apples and then give away 5 apples, how many apples do you have?

7. Biff is a calc 3 student at a large state university and he's having some trouble. Biff says "Dude, how is there such a long list of vocab words for my math class? There's potential, and there's conservative, and there's gradient, and there's this fundament thing. The prof said we're s'posed to be able to explain how they're all connected, but they sound pretty different to me!"

   Explain as clearly as possible to Biff what connection might exist between the various terms he mentioned.

8. Let **F** be the vector field $\mathbf{F}(x, y, z) = 4x\,\mathbf{i} + xz^2\,\mathbf{j} + x^7y^3\,\mathbf{k}$. Let S be the portion of the paraboloid $z = x^2 + y^2$ below $z = 25$, along with the disk $x^2 + y^2 \le 25$ in the plane z = 25, all with outward orientation. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let **F** be the vector field $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + 2\,\mathbf{k}$. Let S be the portion of the cylinder $x^2 + y^2 = 16$ between $z = 0$ and $z = 6$, oriented away from the z-axis. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}$ be the vector field $\mathbf{F} = x\,\mathbf{i} - 10x\,\mathbf{j} + (-z - x)\,\mathbf{k}$. Let S be the portion of the sphere centered at the origin with radius 5 below z = 3, with normal vectors oriented outward. Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

Extra Credit (5 points possible):

If $\mathbf{a}$ is a constant vector, $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and S is an oriented, smooth surface with simple, closed, smooth, positively oriented boundary curve C, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}\,.$$