

## Quiz 2

## Calculus 3

Due by 11/4/20

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Compute  $\int_C \vec{F} \cdot d\vec{r}$  for the vector field  $\vec{F}(x, y) = xy\vec{i} - y\vec{j}$  and with  $C$  a line segment from  $(1, 2)$  to  $(3, 0)$ .

$$\text{I. } \begin{aligned} x(t) &= 1 + 2t \\ y(t) &= 2 - 2t \end{aligned} \quad \vec{r}(t) = \langle 1 + 2t, 2 - 2t \rangle \quad 0 \leq t \leq 1$$

$$\text{II. } \vec{F}(\vec{r}(t)) = \langle (1+2t)(2-2t), -(2-2t) \rangle = \langle (2-2t+4t-4t^2), (-2+2t) \rangle$$

$$\text{III. } \vec{r}'(t) = \langle 2, -2 \rangle \quad \langle (2+2t-4t^2), (-2+2t) \rangle$$

$$\text{IV. } \int_0^1 \langle (2+2t-4t^2), (-2+2t) \rangle \cdot \langle 2, -2 \rangle dt$$

$$\text{V. } \int_0^1 (4+4t-8t^2+4-4t) dt = \int_0^1 (8-8t^2) dt$$

$$\int_0^1 8t - \frac{8}{3}t^3 = 8 - \frac{8}{3} = 5\frac{1}{3} = \frac{16}{3}$$

2. Compute  $\int_C \vec{F} \cdot d\vec{r}$  for the vector field  $\vec{F}(x, y) = \langle 5x^4y^2, 2x^5y \rangle$  and with  $C$  the counterclockwise arc of a circle beginning at  $(2, 0)$  and ending at  $(-\sqrt{2}, \sqrt{2})$ .

potential function  $\rightarrow f(x, y) = x^5 y^2$

evaluated from  $(2, 0)$  to  $(-\sqrt{2}, \sqrt{2})$

$$x^5 y^2 \Big|_{(2, 0)}^{(-\sqrt{2}, \sqrt{2})} = (-\sqrt{2})^5 (\sqrt{2})^2 - (-\sqrt{2})^5 (0)^2 = -8\sqrt{2}$$