Exam 2 Real Analysis 1 11/6/20

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of the lettered problems you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. State the definition of the derivative of a function $f(x)$ at $x = a$.

2. a) State the definition of a set E being closed.

b) State the definition of a set E being open.

3. State the Indermediate Value Theorem.

4. a) State the definition of a compact set.

b) State the Heine-Borel Theorem.

c) Give an example of an open cover for $(0, 2020)$ that has no finite subcover.

5. Prove that if f is differentiable at a then f is continuous at a.

6. State and prove Fermat's Theorem.

7. State and prove Rolle's Theorem.

□ A. State and prove the Product Rule for Derivatives, making clear how your hypotheses are necessary.

□ B. Prove that the product of continuous functions is continuous.

☐ C. State and prove the Boundedness Theorem.

□ D. i.) State the Extreme Value Theorem

ii.) State the Mean Value Theorem

☐ E. Give an example of a function that is defined for all real inputs, but continuous nowhere.

□ F. Let $f(x) = |x| \cdot x$. Find $f'(x)$, or show it does not exist.