**Problem Set 3** **Real Analysis 1** **Due 9/18/20**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. **Submit on Moodle as a pdf.**

1. Show that $\left\{\frac{b-a}{2^n}\right\}$ converges to 0.

2. If a sequence $\{a_n\}$ converges to 0, and a sequence $\{b_n\}$ is bounded, then the sequence $\{a_n b_n\}$ converges to 0.

3. If a sequence $\{a_n\}$ converges to 0, and $\{a_n b_n\}$ converges to zero, then the sequence $\{b_n\}$ is bounded.

4. Determine whether $\lim_{n\to\infty} \frac{1}{n} \sin \frac{1}{n}$ exists, and find its value. [Kosmala 2.2.11(l)]

5. Suppose that the sequence $\{a_n\}$ diverges to $+\infty$. Find examples of sequences $\{a_n\}$ and $\{b_n\}$ so that $\left\{\frac{a_n}{b_n}\right\}$
(a) diverges to $+\infty$.
(b) converges to 7.
(c) converges to 0.
(d) diverges to $-\infty$.
(e) oscillates.

6. Suppose $\{a_n\}$ and $\{b_n\}$ both oscillate. Then $\{a_n \cdot b_n\}$ and $\{a_n + b_n\}$ oscillate.

7. If $\{a_n\}$ is a Cauchy sequence and $S = \{a_n | n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is constant from some point on.

8. Let $s_0$ be an accumulation point of $S$. Then any neighborhood of $s_0$ contains at least one point of $S$ different from $s_0$ iff any neighborhood of $s_0$ contains infinitely many points of $S$.