**Problem Set 4**          **Real Analysis 1**          **Due 9/25/20**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Suppose that $\lim_{x\to\infty} f(x) = A$, where $f$ is a function with domain $D$. Prove (directly from the definition) that if $k$ is a constant, $\lim_{x\to\infty}[kf(x)] = kA$.

2. Suppose that $\lim_{x\to\infty} f(x) = A$, where $f$ is a function with domain $D$. Prove (directly from the definition) that $\lim_{x\to\infty}|f(x)| = |\lim_{x\to\infty} f(x)|$.

3. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where $f$ and $g$ are functions with domain $D$. Prove (directly from the definition) that $\lim_{x\to\infty}[f(x) \cdot g(x)] = A \cdot B$.

4. If a limit exists for $f$ as $x$ approaches $a$, then that limit is unique.

5. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where $f$ and $g$ are functions with domain $D$. If $\forall x \in D, f(x) < g(x)$ then $A < B$.

6. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where $f$ and $g$ are functions with domain $D$. If $\forall x \in D, f(x) \le g(x)$ then $A \le B$.

7. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then $f$ is bounded.

8. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then $f$ is bounded on some set $D_1$, with $D_1 \subseteq D$.