Problem Set 5   **Real Analysis 1**   **Due 10/12/20**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. The sum of continuous functions is continuous.

2. The product of continuous functions is continuous.

3. The function $f(x) = x$ is continuous.

4. Let $c \in \mathbb{R}$. The function $g(x) = c$ is continuous.

5. Suppose that D is the domain of f. If f is continuous at a, then there exists $\delta > 0$ such that f is bounded on the set $(a - \delta, a + \delta) \cap D$.

6. Determine where the function

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

   is continuous.

7. Determine where the function

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   is continuous.

8. Determine where the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   is continuous.

9. fg and f continuous on (a, b) implies that g is continuous on (a, b).

10. $|f|$ continuous on (a, b) implies that f is continuous on (a, b).