Problem Set 8 **Real Analysis 1** **Due 11/20/20**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let f be continuous at $x = c \in [a,b]$. Let x_n be a sequence in $[a,b]$ converging to c. Then the sequence $\{f(x_n)\}$ converges to $f(c)$.

2. Let $f(x) = x$, and let $\mathscr{P}$ be a regular partition of $[a,b]$ with n subdivisions. Evaluate $U(\mathscr{P}, f)$ and $L(\mathscr{P}, f)$, and find their limits as n approaches ∞.

3. Suppose that a function $f : [a,b] \to \mathbb{R}$ is bounded and $\mathscr{P} = x_0, x_1, x_2, ..., x_n$ is a partition of $[a,b]$. Then there exist $m, M \in \mathbb{R}$ such that

$$m(b-a) \leq L(\mathscr{P}, f) \leq S(\mathscr{P}, f) \leq U(\mathscr{P}, f) \leq M(b-a)$$

4. Suppose that a function $f : [a,b] \to \mathbb{R}$ is bounded and $\mathscr{P}$ and $\mathscr{Q}$ are partitions of $[a,b]$. Show that if $\mathscr{P} \subseteq \mathscr{Q}$, then $U(\mathscr{Q}, f) \leq U(\mathscr{P}, f)$.