Exam 1 Calc 1 9/10/2021

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:

a) $\lim_{x\to 3^-} g(x)$

1, because it is as x approaches 3 from the left

b) $\lim_{x\to 3^+} g(x)$

-1, because it is as x approaches 3 from the right Great

c) $\lim_{x\to 3} g(x)$

DNE, because it is asking as x approaches 3 & the limit does not agree on one, it is at 2 different places

d) $\lim_{x\to 1^+} g(x)$

1, because it is as x approaches 1 from the right

e) $\lim_{x\to 1^-} g(x)$

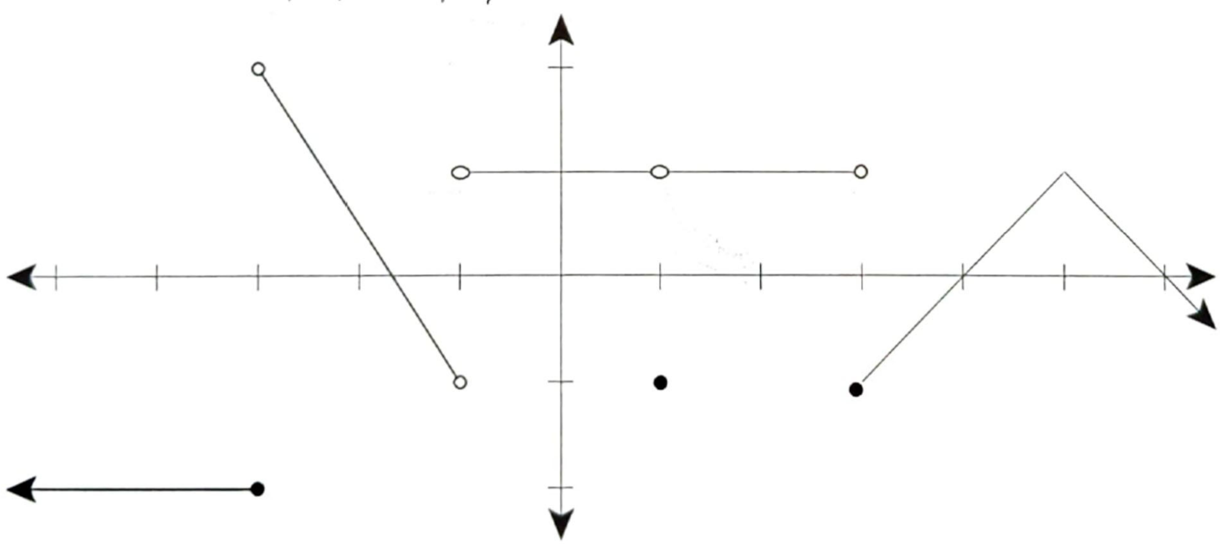
1, because it is as x approaches 1 from the left

2. For which values of x does the function fail to be continuous?

g(x) fails to be continuous at, $x = -3, -1, 1, 3$

$\lim_{x\to -3^-} g(x) \neq \lim_{x\to -3^+} g(x)$ $g(1)$ DNE Good

$\lim_{x\to -1^-} g(x) \neq \lim_{x\to -1^+} g(x)$ $\lim_{x\to 3^-} g(x) \neq \lim_{x\to 3^+} g(x)$



3. Use the table to evaluate the following expressions.

x	1	2	3	4	5	6
$f(x)$	6	3	2	5	1	4
$g(x)$	4	5	1	3	6	2

a) $f(2)$

$\boxed{3}$

b) $f(g(1))$ $g(1) = 4$

$f(4) = \boxed{5}$

c) $g(f(1))$

$f(1) = 6$ $g(6) = \boxed{2}$

Excellent!

d) $f \circ g\,(3) = f(g(3))$

$g(3) = 1$ $f(1) = \boxed{6}$

e) $g \circ f\,(3) = g(f(3))$

$f(3) = 2$ $g(2) = \boxed{5}$

4. Consider the function $f(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. What is $\lim_{x \to 0} f(x)$, and why?

$\lim_{x \to 0} f(x)$ Does not exist because from 0^- $\lim_{x \to 0^-} f(x) = 0$ but from 0^+ $\lim_{x \to 0^+} f(x) = 1$. Since the left and the right don't agree (in this political climate? shocker!) the limit does not exist.

Excellent!

5. Find the limits:

a) $\lim_{x\to 5^+} \frac{x^2-25}{x-5} = \lim_{x\to 5^+} \frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5^+} (x+5) = 5+5 = 10$

Nice!

b) $\lim_{x\to 5^-} \frac{x^2-25}{x-5} = \lim_{x\to 5^-} \frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5^-} (x+5) = 5+5 = 10$

6. Evaluate $\lim_{x\to\infty} \frac{x^2+5}{2x^3+1}$.

$\lim_{x\to\infty} \frac{x^2+5}{2x^3+1}$ divide by highest denom power

$\lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}^{\to 0} + \cancel{\frac{5}{x^3}}^{\to 0}}{2+\cancel{\frac{1}{x^3}}^{\to 0}} = \frac{0}{2}$ or 0

Excellent

7. Jordan is a calculus student at Enormous State University, and there's some trouble. Jordan says "Wow. We had to do this group assignment in Calc today and it got kind of out of control. My group had these two people, Biff and Bunny, and they really just couldn't agree on anything. We were doing one of those average velocity things, you know, like with really smaller and smaller intervals? And Biff got positive average velocity every time he tried, but Bunny got negative every time she tried. I doublechecked them, and we even got one of the TAs to check, and they said the calculation stuff was all right. The TA said this cryptic thing, like about close to positive and negative both. How could that even be?"

Help Jordan by explaining as clearly as you can what the limit must have been, and why.

Most likely the limit was 0, this is because since they were getting both a positive and a negative answer as the average velocity and the TA confirmed the calculations were right, the graph would probably look something like this  and they are most likely working to find the average velocity at close to the peak of its height when the average velocity would go from positive to 0 to negative in a relatively short period of time.

Excellent!

8. A bear drops a watermelon from a tall tree, hoping to scare away a vicious cat. The height of the watermelon t seconds after being dropped is given by $h(t) = -16t^2 + 36$ for values of t between 0 and 1.5.

Give answers accurate to at least 4 digits.

a) Find the average velocity of the watermelon over the interval [1, 1.5].

$$\text{Average velocity} = \frac{h(1{,}5) - h(1)}{1{,}5 - 1} = \frac{-16(1{,}5)^2 + 36 - (-16(1)^2 + 36)}{0{,}5}$$

$$= \frac{0 - (20)}{0{,}5} = \underline{-40}$$

b) Find the average velocity of the watermelon over the interval [1.4, 1.5].

$$\text{Average velocity} = \frac{h(1{,}5) - h(1{,}4)}{1{,}5 - 1{,}4} = \frac{-16(1{,}5)^2 + 36 - (-16(1{,}4)^2 + 36)}{0{,}1}$$

$$= \frac{0 - (4{,}64)}{0{,}1} = \underline{-46{,}4}$$

c) Find the average velocity of the watermelon over the interval [1.49, 1.5].

$$\text{Average velocity} = \frac{h(1{,}5) - h(1{,}49)}{1{,}5 - 1{,}49}$$

$$= \frac{-16(1{,}5)^2 + 36 - (-16(1{,}49)^2 + 36)}{0{,}01}$$

$$= \frac{0 - 0{,}4784}{0{,}01}$$

$$= \underline{-47{,}84}$$

Good

9. Let $f(x) = 5 - 4x - x^2$. Evaluate the difference quotient $\dfrac{f(3+h) - f(3)}{h}$.

• $\dfrac{f(3+h) - f(3)}{h}$

we have $f(3) = 5 - 4(3) - 3^2 = -16$

so : $\dfrac{f(3+h) - f(3)}{h}$

$= \dfrac{5 - 4(3+h) - (3+h)^2 - f(3)}{h}$

$= \dfrac{5 - 12 - 4h - (h^2 + 6h + 9) - f(3)}{h}$

$= \dfrac{\cancel{-7} - 4h - h^2 - 6h \cancel{-9} \cancel{-(-16)}}{h}$

$= \dfrac{-h^2 - 10h}{h}$

$= \dfrac{\cancel{h}(-h - 10)}{\cancel{h}}$

$= -h - 10$

Excellent!

10. Let $f(x) = mx + b$. Evaluate the difference quotient $\dfrac{f(3+h)-f(3)}{h}$.

$$\frac{f(3+h)-f(3)}{h}$$

$$= \frac{m(3+h)+b-(m\times 3+b)}{h}$$

$$= \frac{\cancel{3m}+mh+\cancel{b}-\cancel{3m}-\cancel{b}}{h}$$

$$= \frac{mh}{h}$$

$$= m$$

Nice!