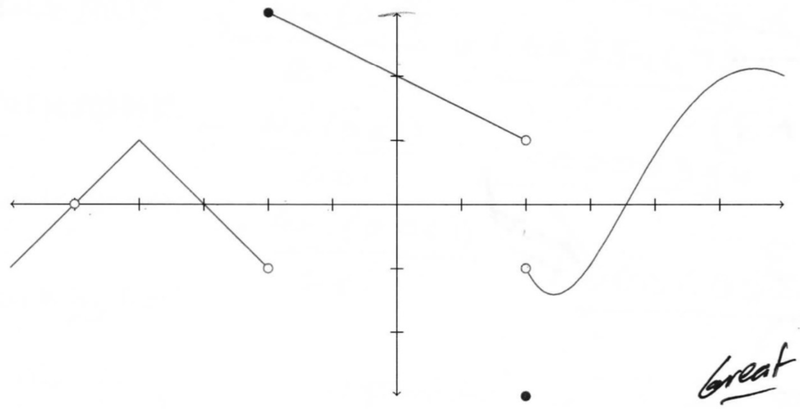
Exam 1a **Calc 1** **9/16/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



Great

1. (a) What is $\lim_{x\to 2^+} f(x)$? -1, the limit as it approaches 2 from the right

   (b) What is $\lim_{x\to 2^-} f(x)$? 1, the limit as it approaches 2 from the left

   (c) What is $\lim_{x\to 2} f(x)$? DNE, The limit does not agree as both sides approach

   (d) What is $\lim_{x\to -2^+} f(x)$? 3, the limit as it approaches -2 from the right

   (e) What is $\lim_{x\to -2^-} f(x)$? -1, the limit as it approaches -2 from the left

2. For which values of x does the function fail to be continuous?

x = -5, as the function does not exist but the limit = 0
x = -2, as the limit does not exist.
x = 2, as the limit does not exist.

Great

3. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$.

factor out

$= \lim_{x \to 3} \frac{(x-\cancel{3})(x+3)}{x-\cancel{3}}$

$= \lim_{x \to 3} x+3$

$= 3+3 \boxed{= 6}$

Good!

4. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	5	4	6	2	3	1
$g(x)$	6	1	2	3	4	5

(a) $f(2)$ $= 4$

(b) $f(g(3))$ $= f(2) = 4$

(c) $g(f(3))$ $= g(6) = 5$

(d) $(f \circ g)(1)$ $= f(g(1)) = f(6) = 1$

(e) $(g \circ f)(1)$ $= g(f(1)) = g(5) = 4$

Great!

5. Let $f(x) = \frac{\tan x}{x}$. **Make sure your calculator is in radian mode.** Give answers accurate to at least 8 decimal places.

(a) What is $f(0.2)$? $\frac{\tan(0.2)}{0.2} = 1.013550178$

(b) What is $f(0.1)$? $\frac{\tan(0.1)}{0.1} = 1.003346721$

(c) What is $f(0.01)$? $\frac{\tan(0.01)}{0.01} = 1.000033335$

(d) What is $f(0.001)$? $\frac{\tan(0.001)}{0.001} = 1.000000333$

(e) What is $\lim_{x\to 0} f(x)$?

$\lim_{x\to 0} f(x) = 1$ because each point continues to get closer and closer to 1

Great!

6. Evaluate the following limits:

(a) $\lim_{x\to\infty} \frac{9}{e^x - 7} = \boxed{0}$

e to a really big power is going to get closer to infinity. So the denominator will get larger + larger, making the whole function get really close to zero as x approaches ∞.

(b) $\lim_{x\to-\infty} \frac{9}{e^x - 7} = \boxed{-\frac{9}{7}}$

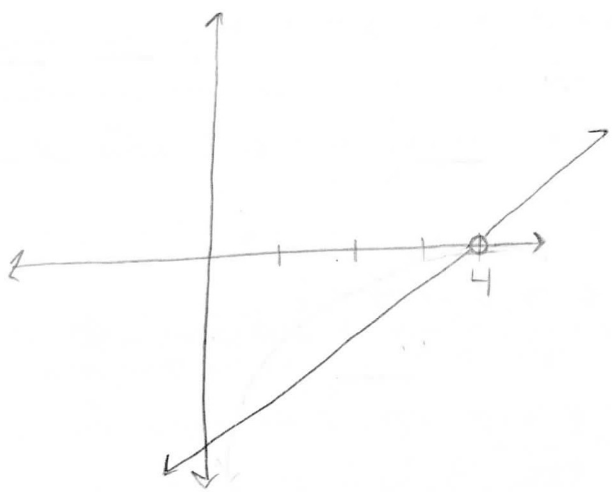
e to an increasingly negative power will be close to zero or zero? (depending on how big of a number you're using), so that value becomes inconsequential. You're left with whatever is left after e^x zeroes out.

Yes

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. Calc is totally killing me. I thought I knew it all from high school, but now there's all this limit crap and you can't just put it in your calculator once, they say you gotta try a bunch of things. So like, I tried 3.9 and 3.99 for approaching 4 on this quiz, and every answer I got was negative, right, so you know whatever the limit is it has to be negative. But this girl next to me said it was zero, but that can't be, right, since negatives never get to zero!"

   Help Biff by explaining as clearly as you can whether the limit he's trying to find could be zero.

The graph could look something like this:



In this image, the values approach zero, but they are approaching from the negative side of the x-axis (zero). The limit could still be zero, but the function is approaching zero from the negative side + becoming more positive.

Excellent!

8. Over the first few seconds after a chunk of frozen urine falls off the vent of a Russian surveilance plane flying over Ukraine, the height (in feet) of the chunk is given by the function $h(t) = 30,000 - 16t^2$. What is the chunk's average velocity over the time period beginning when $t = 2.5$ and lasting

(a) 0.5 seconds

$$\text{average velocity} = \frac{30000 - 16(3)^2 - 30000 + 16(2.5)^2}{3 - 2.5}$$

$$= \underline{-88}$$

(b) 0.1 seconds

$$\text{average velocity} = \frac{30000 - 16(2.6)^2 - 30000 + 16 \times (2.5)^2}{2.6 - 2.5}$$

$$= \underline{-81.6}$$

(c) 0.01 seconds

$$\text{average velocity} = \frac{30000 - 16 \times (2.51)^2 - 30000 + 16 \times (2.5)^2}{2.51 - 2.5}$$

$$= \underline{-80.16}$$

Good

9. Given the function $f(x) = x^3$, simplify $\frac{f(a+h)-f(a)}{h}$.

$f(a+h) = (a+h)^3 = (a+h)(a+h)(a+h) = (a^2+2ah+h^2)(a+h)$

$= a^3+3a^2h+3ah^2+h^3$

$f(a) = a^3$

$\frac{\cancel{a^3}+3a^2\cancel{h}+3ah^{\cancel{2}}+h^{\cancel{3}}-\cancel{a^3}}{h} = 3a^2+3ah+h^2$

Great

10. (a) Evaluate $\lim_{x\to 25} \frac{5-\sqrt{x}}{25x-x^2}$.

Nice!

$$= \lim_{x\to 25} \frac{(5-\sqrt{x})(5+\sqrt{x})}{(25x-x^2)(5+\sqrt{x})}$$

$$= \lim_{x\to 25} \frac{25-\cancel{x}}{x(25-\cancel{x})(5+\sqrt{x})}$$

$$= \lim_{x\to 25} \frac{1}{x(5+\sqrt{x})}$$

$$= \frac{1}{250}$$

Yes

(b) For what value(s) of a will $\lim_{x\to a^2} \frac{a-\sqrt{x}}{a^2x-x^2}$ be 1/100?

$$\lim_{x\to a^2} \frac{a-\sqrt{x}}{a^2x-x^2}$$

$$= \lim_{x\to a^2} \frac{(a-\sqrt{x})(a+\sqrt{x})}{x(a^2-x)(a+\sqrt{x})}$$

$$= \lim_{x\to a^2} \frac{a^2-\cancel{x}}{x(a^2-\cancel{x})(a+\sqrt{x})}$$

$$= \lim_{x\to a^2} \frac{1}{x(a+\sqrt{x})}$$

Excellent!

$$= \frac{1}{a^2(a+a)}$$

$$= \frac{1}{2a^3}$$

if it will be $\frac{1}{100}$

$$a^3 = 50$$

$$a = \sqrt[3]{50}$$