**Exam 4b** **Calc 1** **11/11/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Find the critical numbers of $f(x) = x^3 - 12x^2 + 36x$.

$f(x) = x^3 - 12x^2 + 36x$

I) $f'(x) = 3x^2 - 24x + 36$

$= 3(x^2 - 8x + 12)$

II) $0 = 3(x^2 - 2x - 6x + 12)$

$= 3(x(x-2) - 6(x-2))$

$= 3(x-6)(x-2)$

where, $x = 6$, and, $x = 2$

Excellent!

The critical numbers are 6 and 2

2. Find the $x$ coordinates of any inflection point(s) of $f(x) = x^3 - 12x^2 + 36x$.

inflection is where concavity changes

I $f(x) = x^3 - 12x^2 + 36x$

$f'(x) = 3x^2 - 24x + 36$

II $f''(x) = 6x - 24$

$0 = 6x - 24$

$6x = 24$

$x = 4$

Great

| int | $f''(x)$ |
|---|---|
| $-\infty, 4$ | $-$ |
| $4, \infty$ | $+$ |

$y = 4^3 - 12(4)^2 + 36(4)$

$y = 16$

10

$IP = (4, 16)$

3. Find the most general antiderivatives of the the following functions:

(a) $f(x) = x^n$

$\vec{F}(x) = \frac{1}{n+1} \cdot x^{n+1} + C$

(b) $f(x) = \sin x$

$F(x) = -\cos x + C$

(c) $f(x) = e^x$

$F(x) = e^x + C$

(d) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$

$F(x) = \arcsin x + C$

(e) $f(x) = \dfrac{1}{x}$

$F(x) = \ln|x| + C$

4. Let $f(x) = x^2 - 4x + 3$. Find the absolute maximum and absolute minimum values of $f$ on $[0, 5]$.

$f'(x) = 2x - 4 = 0$

$2(x-2) = 0$

$x - 2 = 0$
$+2 \quad +2$

$x = 2$

$f(0) = 3$

$f(2) = -1 \rightarrow$ minimum

Excellent!

[redacted] $\rightarrow$ maximum

$^\wedge f(5) = 8$

5. Let $f(x) = 6x - 2x^3$. Find the largest possible interval(s) where $f$ is decreasing.

$f'(x) = 6 - 6x^2$

$0 = 6(1 - x^2)$

$\Rightarrow x^2 = 1$

$x = \pm 1$

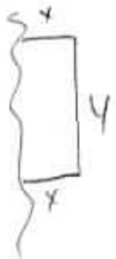
$\infty \quad -1 \quad 0 \quad 1 \quad \infty$

| Intervals | $f'(x)$ |
|---|---|
| $(-\infty, -1)$ | $-$ |
| $(-1, 1)$ | $+$ |
| $(1, \infty)$ | $-$ |

Excellent!

The largest possible intervals where $f$ is decreasing are $(-\infty, -1)$ & $(1, \infty)$.

6. [Stewart] A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?




$2400 = y + x + x$

$2400 = y + 2x$

$2400 - 2x = y$

$A = x \cdot y$

$A = x \cdot (2400 - 2x)$

$A = 2400x - 2x^2$

$A' = 2400 - 4x$

$0 = 2400 - 4x$

$\frac{-2400}{-4} = \frac{-4x}{-4}$

$600 = x$

$2400 = y + 600 + 600$
$-1200$

$1200 = y$

Field can be 600 ft by 1200 ft

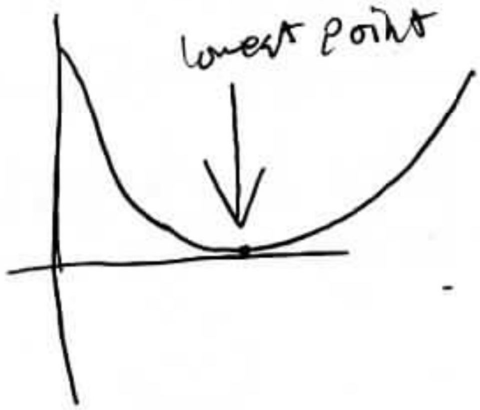
Great Job!

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so confusing? I get the slopey parts, you know? But then they have this cavity part, which makes no sense because that's teeth, right? But so somehow the cavity tells you a max instead of a min or something, right? What's up with that?"

Help Bunny by explaining as clearly as you can how concavity connects to maxes and mins.

OMG!! Dont worry Bunny, I have a dentist friend who has taken calc, he can help. The concavity is related to how the slope is changing. If the slope is negative but getting less negative, then you would be aproaching a point that will be the lowest of an area. This also works inversly with maxes.

concave up graph




Excellent.

8. Let $y = \frac{1}{2}x - \sin x$. Find the exact coordinates of the lowest point on this graph in the interval $[0, 2\pi]$.

$$y' = \frac{1}{2} - \cos x$$

$$0 = \frac{1}{2} - \cos x$$

$$\cos x = \frac{1}{2}$$

$$x = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$$

And when $x = \frac{\pi}{3}$, $y = \frac{1}{2}\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)$

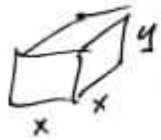
$$= \frac{\pi}{6} - \frac{\sqrt{3}}{2}$$

So $\left(\frac{\pi}{3}, \frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$ is the lowest point

$\approx (1.047, -0.342)$ is pretty close

9. Rectangular storage bins are to be made with square bases and open tops. The volume of each bin is to be 1 cubic meter. What dimensions use the least (in terms of square meters) amount of material?

Volume: $x \cdot x \cdot y = 1 \Rightarrow y = \frac{1}{x^2}$




Area: $\text{Area} = \overbrace{x \cdot x}^{\text{base}} + \overbrace{4x \cdot y}^{\text{sides}}$

$$A(x) = x^2 + 4x\left(\frac{1}{x^2}\right)$$

$$A(x) = x^2 + 4x^{-1}$$

$$A'(x) = 2x - 4x^{-2}$$

$$0 = 2x - \frac{4}{x^2}$$

$$0 = 2x^3 - 4$$

$$0 = x^3 - 2$$

$$2 = x^3$$

$$x = \sqrt[3]{2}$$

So $y = \frac{1}{(\sqrt[3]{2})^2} = 2^{-2/3}$

So the box should be $\sqrt[3]{2}$ by $\sqrt[3]{2}$ by $\frac{1}{\sqrt[3]{2^2}}$

10. [Anton] Suppose that the population of oxygen-dependant bacteria in a pond is modeled by the equation

$$P(t) = \frac{60}{5 + 7e^{-t}}$$

where $P(t)$ is the population (in billions) $t$ days after an initial observation at time $t = 0$. What can you say about when the population is at a maximum?

$$P'(t) = \frac{0 \cdot (5 + 7e^{-t}) - 60 \cdot -7e^{-t}}{(5 + 7e^{-t})^2}$$

$$0 = \frac{420e^{-t}}{(5 + 7e^{-t})^2}$$

$$0 = 420e^{-t}$$ This is never true!

There are <u>no</u> places where the population is a maximum!