Exam 1 **Calc 3** **9/30/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Show that the function $f(x, y) = \dfrac{x^2}{x^2 + y^2}$ fails to have a limit at $(0, 0)$.

3. Suppose that $u = f(x, y)$, where $x = x(r, s, t), y = y(r, s, t)$. Write the Chain Rule formula for $\frac{\partial u}{\partial s}$. Make very clear which derivatives are partials.

4. Find an equation for the plane tangent to $f(x, y) = xe^{-y}$ at the point $(2, 1)$.

5. Let $f(x, y) = \sin x + \cos y$.

(a) Find the directional derivative of f in the direction of the vector $\vec{v} = \langle -3, 4 \rangle$ at the point $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.

(b) In which dirction is the directional derivative greatest at the point $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$ the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This directional derivative crap is totally killing me. How the heck do derivatives have directions now anyway? So they told us that there was this vector that pointed the direction of greatest increase from the origin, right? And it pointed one comma one, so up and right. But then they said the max was at one comma two, which is impossible, because steepest was toward one comma one instead, right? So I figured they just had a typo, so I crossed out the two and put one."

   Help Biff by explaining as clearly as you can whether the information he was given could be consistent, and how you know.

8. [Anton 6th] Find and classify all critical points of $f(x, y) = x^3 - 3xy - y^3$.

9. [Stuart] A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of cardboard used.

10. Consider the parabola $z = x^2 + y^2$.

(a) Find an equation for the line normal to the parabola at the point (a, b).

(b) At what z value(s) does this normal line intersect the surface?

Extra Credit (5 points possible): For which values of a does $f(x, y) = x^3 - 3xy - y^3 + ax$ have a local maximum?