Problem Set 6 **Real Analysis 1** **Due 10/21/22**

Ten of these problems will be graded, with each problem worth 2 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. If $E \subseteq \mathbb{R}$ is closed, then $\mathbb{R} \backslash E$ is open.
2. If $\mathbb{R} \backslash E$ is open, then $E \subseteq \mathbb{R}$ is closed.
3. Show that if f is continuous at $c \in [a, b]$ then for any sequence $\{x_n\}$ (with $x_n \in E, \forall n \in \mathbb{N}$) converging to c, the sequence $\{f(x_n)\}$ converges to $f(c)$.
4. Give an example of a function f which is continuous on a bounded set D, but not bounded on D.

For the following, prove or give a counterexample:

5. If a function f is bounded and continuous on D, then f is uniformly continuous on D.
6. If a function f is continuous at a real number $x = a$, then $[f(x)]^2$ is also continuous at $x = a$.
7. The set $\{1, 2, 3\}$ is closed.
8. The set $\left\{\frac{1}{n} | n \in \mathbb{N}\right\}$ is closed.
9. The set $(0, \infty)$ is open.
10. A composition of two continuous functions is continuous