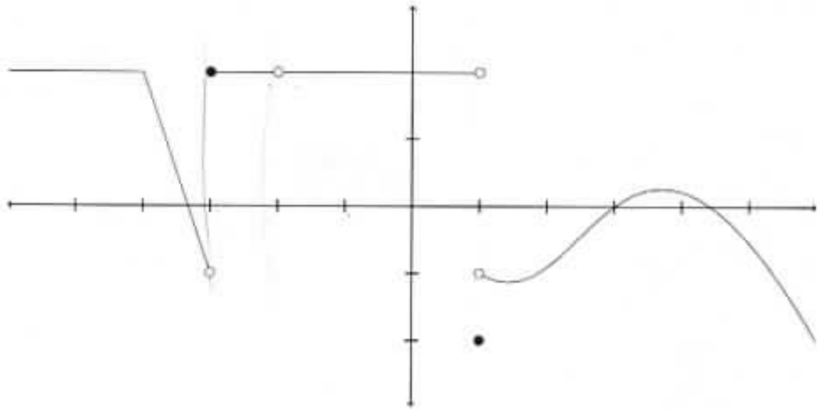
Exam 1 **Calc 1** **9/15/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



1. (a) What is $\lim_{x\to 1^+} f(x)$? $= -1$

   Excellent!

   (b) What is $\lim_{x\to 1^-} f(x)$? $= 2$

   (c) What is $\lim_{x\to 1} f(x)$? = DNE, because left and right are different

   (d) What is $\lim_{x\to -2^+} f(x)$? $= 2$

   (e) What is $\lim_{x\to -2^-} f(x)$? $= 2$

   (f) What is $\lim_{x\to -2} f(x)$? $= 2$ because left and right are the same

2. Use interval notation to indicate where the function above is continuous.

$(-\infty,-3)\cup(-3,-2)\cup(-2,1)\cup(1,\infty)$ discontinuous @ $x=-3$, $x=-2$, $x=1$

Great

[illegible]

3. Evaluate $\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$.

$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x-1)}{\cancel{x-5}}$

$= \lim_{x \to 5} x - 1$

$= 5 - 1$

$= 4$

Great

4. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	5	4	6	1	3	2
$g(x)$	1	6	2	3	5	4

(a) $f(5)$ $= \underline{3}$

(b) $f(2) + 1$ $= \underline{4} + 1 = \underline{5}$

Great

(c) $f(2 + 1)$ $= \underline{6}$

(d) $(f \circ g)(4)$ $= f(g(4)) = f(3) = \underline{6}$

(e) $(g \circ f)(4)$ $= g(f(4)) = g(1) = \underline{1}$

5. (a) Evaluate $\lim_{x\to\infty} \frac{3x}{x-5}$.

$$\lim_{x\to\infty} \frac{3x \cdot \frac{1}{x}}{x-5 \cdot \frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{3}{1-\frac{5}{x}} \Rightarrow \underline{\frac{3}{1}}$$

(b) Evaluate $\lim_{x\to 5^-} \frac{3x}{x-5}$.

$$\lim_{x\to 5^-} \left(3x \cdot \frac{1}{x-5}\right)$$

$$(15)(-\infty) \Rightarrow \underline{-\infty}$$

Good

(c) Evaluate $\lim_{x\to 5^+} \frac{3x}{x-5}$.

$$\lim_{x\to 5^+} (3x)\left(\frac{1}{x-5}\right)$$

$$(15)(\infty) \Rightarrow \underline{\infty}$$

6. A huge pumpkin is catapulted off the top of Murray Hall, and its height in feet after t seconds is given by $h(t) = -16t^2 + 64t + 100$. Find the average velocity of the pumpkin over the

(a) first 1 second of flight

$$h(0) = -16(0)^2 + 64(0) + 100 = 100$$

$$h(1) = -16(1)^2 + 64(1) + 100 = 148$$

$$\frac{h(0)-h(1)}{0-1} = \frac{100-148}{0-1} = \frac{-48}{-1} = 48 \text{ ft/s}$$

Good

(b) first 0.1 seconds of flight

$$\frac{h(0)-h(.1)}{0-.1} =$$

$$h(0) = 100$$

$$h(.1) = -16(.1)^2 + 64(.1) + 100 = 106.24$$

$$= \frac{-6.24}{-.1} = 62.4 \text{ ft/s}$$

7. Samantha is a calculus student at Enormous State University, and she's having some trouble with limits. Samantha says "So I totally bombed this quiz we had about limits. We were supposed to say what the limit of $\frac{n-1}{n}$ was, and so I said I didn't really know, but for sure it had to be less than 1, because stuff like $\frac{1}{2}$ and $\frac{2}{3}$ and all that are less than 1. So whatever the limit is, it's gotta be less than 1 too, right? But the TA didn't really like it, I guess, and he wrote this long note I totally couldn't even read, and I got no points, so I really better figure this out for the exam, huh?"

Help Samantha by explaining, in terms she can understand, either how to convince her professor she's right, or how it is that terms less than 1 can have a limit equal to 1.

"That sounds like a tough question, but I think you were on the right track! I learned about that in my calc class too, and if you keep putting <u>bigger</u> numbers in, that function gets <u>bigger</u>. You used 1/2 and 2/3, and if you kept going you'd eventually hit 999/1000, and 9,999/10,000. Yes the function never hits 1, you're right about that part, but since it gets infinitely close to 1, the limit is 1."

Excellent!

8. Evaluate $\lim_{x\to\infty}\left(\sqrt{x}-\sqrt{x+2}\right)$

$$= \lim_{x\to\infty} \sqrt{x} - \sqrt{x+2}$$

$$= \lim_{x\to\infty} \sqrt{x} - \sqrt{x+2} \times \frac{\sqrt{x}+\sqrt{x+2}}{\sqrt{x}+\sqrt{x+2}}$$ [∵ Rationalizing to simplify)

$$= \lim_{x\to\infty} \frac{(x)-(x+2)}{\sqrt{x}+\sqrt{x+2}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}-\cancel{x}-2}{\sqrt{x}+\sqrt{x+2}}$$

$$= \lim_{x\to\infty} \frac{-2}{\sqrt{x}+\sqrt{x+2}}$$

$$= \lim_{x\to\infty} \frac{\frac{-2}{\sqrt{x}}}{\sqrt{\frac{\cancel{x}}{\cancel{x}}}+\sqrt{\frac{x+2}{x}}}$$

$$= \lim_{x\to\infty} \frac{\frac{-2}{\sqrt{x}} \to 0}{\sqrt{1}+\sqrt{1+\frac{2}{x}\to 0}}$$

Excellent!

$$= \frac{0}{1+1}$$

$$= 0 \quad //$$

9. [Rogawski/Adams 3rd] According to the Michaelis-Menten equation, when an enzyme is combined with a substrate of concentration s (in millimolars), the reaction rate (in micromolars/min) is

$$R(s) = \frac{As}{K + s}$$

where A and K are constants. Find

$$\lim_{s \to \infty} R(s)$$

$$= \lim_{s \to \infty} \frac{As}{K+s}$$

$$= \lim_{s \to \infty} \frac{\frac{As}{s}}{\frac{K}{s} + \frac{s}{s}}$$

$$= \lim_{s \to \infty} \frac{A}{0+1}$$

$$= A$$

Excellent!

10. Give a formula for a function for which $\lim_{x \to 3^-} f(x) = +\infty$, $\lim_{x \to 3^+} f(x) = +\infty$, and $\lim_{x \to \infty} f(x) = 2$.

$$\lim_{\substack{x \to 3^- \\ 2.999}} \frac{2x}{|x-3|} = \lim_{x \to 3^-} \frac{+}{\underset{\to 0}{+}} = +\infty$$

That works!

$$\lim_{\substack{x \to 3^+ \\ 3.001}} \frac{2x}{|x-3|} = \lim_{x \to 3^+} \frac{+}{\underset{\to 0}{+}} = +\infty$$

$$\lim_{x \to \infty} \frac{2x}{|x-3|} = \lim_{x \to \infty} \frac{2}{(1 - \frac{3}{x})} = 2$$