Exam 2 **Calc 1** **10/6/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.

2. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	5	4	6	1	3	2
$g(x)$	1	6	2	3	5	4
$f'(x)$	2	3	4	5	6	1
$g'(x)$	2	7	3	13	11	8

   (a) If $h(x) = f(x) \cdot g(x)$, what is $h'(2)$ and why?

   (b) If $h(x) = \frac{f(x)}{g(x)}$, what is $h'(5)$ and why?

   (c) If $h(x) = f(g(x))$, what is $h'(3)$ and why?

3. If $f(x) = x^4 + \frac{1}{x^2} + \sqrt{x} + e^x$, what is $f'(x)$?

4. Show that if $f(x) = mx + b$ for some constants m and b, then $f'(x) = m$.

5. Show why the derivative of $\tan x$ is $\sec^2 x$.

6. State and prove the Quotient Rule for derivatives.

7. Biff is a calculus student at Enormous State University, and he's having some trouble with derivatives. Biff says "Dude, I think calculus is broken! Our TA said that this one problem, like with the e to x thing over x squared, right? He said that instead of doing the quotient rude thing on it, you could do it by the product rude thing. Obviously that's wacked, because what I know for sure is that in math there's just one right way to do things, right?"

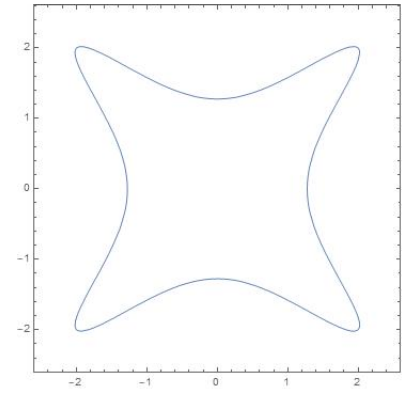
   Help Biff by explaining, in terms he can understand, either how there can be the two different approaches his TA mentioned, or why there can't be.

8. (a) Find the linearization $L(x)$ of the function $f(x) = \sqrt[3]{x}$ at $x = 27$.

(b) Use your linearization from part a to approximate $\sqrt[3]{28}$.

9. A 20 foot ladder is leaning against a wall. If the top slips down the wall at a rate of 3 ft/s, how fast will the foot be moving away from the wall when the top is 16 feet above the ground?

10. (a) Find the slope of the line tangent to $6x^4 - 11x^2y^2 + 6y^4 = 16$ at the point $(2, -2)$.



(b) Show that the slope of the line tangent to $6x^4 - bx^2y^2 + 6y^4 = 16$ at a point of the form $(a, -a)$ does not depend on the value of a.

Extra Credit (5 points possible):
Use the definition of the derivative to compute the derivative of $f(x) = \sqrt{x+7}$.