Exam 1 **Calc 3** **9/29/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Show that the function $f(x, y) = \dfrac{xy}{x^2 + y^2}$ fails to have a limit at $(0, 0)$.

3. Suppose that $u = f(x, y)$, where $x = x(r, s, t), y = y(r, s, t)$. Write the Chain Rule formula for $\frac{\partial u}{\partial s}$. Make very clear which derivatives are partials.

4. Find the directional derivative of $f(x, y) = y^3 - x^2y$ at the point $(3, 5)$ in the direction of the vector $\langle 3, -4 \rangle$.

5. Let $f(x, y) = \sin(x + 2y)$. In which dirction is the directional derivative greatest at the point $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$ the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this is the most totally confusing experience in my life. The professor told us there were these things we definitely had to know for the test, like in my notes I have that she said that the level curvy things are ninety degrees from the direction of greatest increase. And she said we have to know why that's true, but I totally don't have a clue. I looked in the book and it makes no sense at all. She never said anything about it in class, just during the review. So how am I supposed to know why it's true? This is so unfair!"

   Explain clearly to Bunny how she could deduce such a conclusion from other things which she should indeed know.

8. Find and classify all critical points of $f(x, y) = xy(1 - 6x - 8y)$.

9. Find all extrema of the function $f(x, y) = xy^2 - x^3$ subject to the constraint $x^2 + y^2 = 1$.

10. [Stewart] Show that the equation of the tangent plane to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point (x_0, y_0, z_0) can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$$

Extra Credit (5 points possible): In class we noted that something strange occurs with the formulas for the partial derivatives of the function $f(x, y) = \cos\sqrt{x^2 + y^2}$ at (0,0). What can you say about what these derivatives really are?