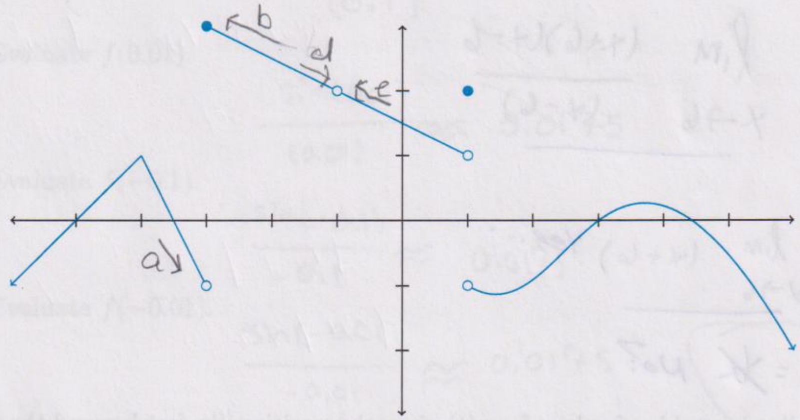
Exam 1a **Calc 1** **9/12/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



1. (a) What is $\lim_{x \to -3^-} f(x)$? $= -1$

   (b) What is $\lim_{x \to -3^+} f(x)$? $= 3$

   Excellent!

   (c) What is $\lim_{x \to -3} f(x)$? DNE because the limit from the left and right don't match $\lim_{x \to -3^-} f(x) \neq \lim_{x \to -3^+} f(x)$

   (d) What is $\lim_{x \to -1^-} f(x)$? $= 2$

   (e) What is $\lim_{x \to -1^+} f(x)$? $= 2$

   (f) What is $\lim_{x \to -1} f(x)$? $= 2$ because $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

2. Use interval notation to indicate where the function above is continuous.

$(-\infty, -3) \cup (-3, -1) \cup (-1, 1) \cup (1, +\infty)$

Great

3. Evaluate $\lim_{x\to 6} \frac{x^2-36}{x-6}$.

$$= \lim_{x \to 6} \frac{(x+6)(x-6)}{x-6}$$ factor

Excellent!

$$= \lim_{x \to 6} \frac{x+6}{1}$$ cancel $x-6$

$= 12$ plug and solve

4. The following table of values for $p(t)$ gives the position (in feet beyond the point where the car was at time 0) of a car t seconds after the driver hits the brakes because they saw a glowing purple giraffe.

t	$p(t)$
0.0	0
0.25	14
0.5	26
0.75	37
1.0	45

$p(.5) = 26$ $p(1) = 45$

(a) What is the car's average velocity over the period from $t = 0.5$ to $t = 1.0$?

$$\frac{p(.5) - p(1)}{.5 - 1} = \frac{-19}{-.5} = 38 \text{ ft/s}$$

(b) What is the car's average velocity over the period from $t = 0.75$ to $t = 1.0$?

Excellent

$$\frac{p(.75) - p(1)}{.75 - 1} = \frac{37-45}{-.25} = \frac{-8}{-.25}$$

$$= 32 \text{ ft/s}$$

5. Let $f(x) = \dfrac{\sin x}{x}$. **Make sure your calculator is in radian mode.**

(a) Evaluate $f(0.1)$.

$f(x) = \dfrac{\sin(.1)}{(.1)} = 0.998334166$

(b) Evaluate $f(0.01)$.

$f(x) = \dfrac{\sin(.01)}{(.01)} = 0.999983333$

(c) Evaluate $f(-0.1)$.

$f(x) = \dfrac{\sin(-.1)}{(-.1)} = 0.998334166$

(d) Evaluate $f(-0.01)$.

$f(x) = \dfrac{\sin(-.01)}{(-.01)} = 0.999983333$ Great

(e) What's $\lim_{x\to 0} f(x)$?

$\lim_{x \to 0} = 1$ Both Sides of the limit were approaching 1 meaning it is 1

6. [WeBWorK] A ball is thrown into the air by a baby alien on a planet in the system of Alpha Centauri with a velocity of 30 ft/s. Its height in feet after t seconds is given by $h(t) = 30t - 27t^2$. Find the average velocity for the time period beginning when $t_0 = 1$ seconds and lasting for 0.1 seconds.

$h(1) = 30(1) - 27(1)^2 = 3$

$h(1.1) = 30(1.1) - 27(1.1)^2 = 0.33$

$\dfrac{0.33 - 3}{1.1 - 1} = \boxed{-26.7 \text{ ft/s}}$

Nice

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Whoa, Calc is kicking my butt! I can plug numbers into functions, you know? But now there's this limit stuff and it's totally confusing. Like, the professor was saying that this function had a limit of 7 when x was 0, but I plugged in like every number and it never gave out 7. That has to mean the limit isn't 7, right?"

Help Biff by explaining, in terms he can understand, whether the reasons he states are good ones for concluding that the limit isn't 7.

Because the limit means that as x approaches 0 The function approaches 7. but this doesn't mean that the function has an out put of 7 for any number. it simply meas as we get really close to x=0 the function is trending towards 7. We never actually hit x=0 on the limit we just get infinitly close to it. this means that the limit can be 7 while the actual output for x=0 can be something very different. the limit is what the function is trending towards not what the actual output is.

Good

8. Consider the function defined by

$$g(x) = \begin{cases} \overset{3\ -2}{3x^2 - 2}, & x \le 1 \\ \underset{5-2}{5 - 2x}, & x > 1 \end{cases} \quad \lim_{x\to 1^-} g(x) = 1 \quad \lim_{x\to 1^+} g(x) = 3 \qquad \lim_{x\to 1} g(x) = DNE$$

Is $g(x)$ continuous at $x = 1$? Explain clearly why or why not.

g(x) is not continuous at x=1 because the limit as x approaches 1 from the left does not equal the limit as x approaches 1 from the right. Therefore, the limit of g(x) as x approaches 1 does not exist. If the limit does not exist, then the function is automatically discontinuous at that point.

Excellent!

$(a+h)(a+h)$

$a^2+ah+ah+h^2$

9. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$.

$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 - \cancel{a^2}}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2a+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2a + h$$

$$= \lim_{h \to 0} 2a + 0$$

$$= \boxed{2a}$$

Wonderful!

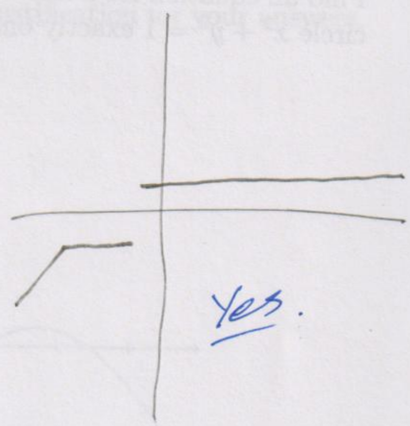
10. Evaluate $\lim_{x \to -1} \frac{|x+1|}{x+|x+2|}$. Be sure to include clear justification.

$\lim_{x \to -1} \frac{|x+1|}{x+|x+2|}$ Does not exist

$\lim_{x \to -1^+} \frac{|x+1|}{x+|x+2|} = \frac{1}{2}$

$\lim_{x \to -1^-} \frac{|x+1|}{x+|x+2|} = -\frac{1}{2}$

Don't match



Yes.