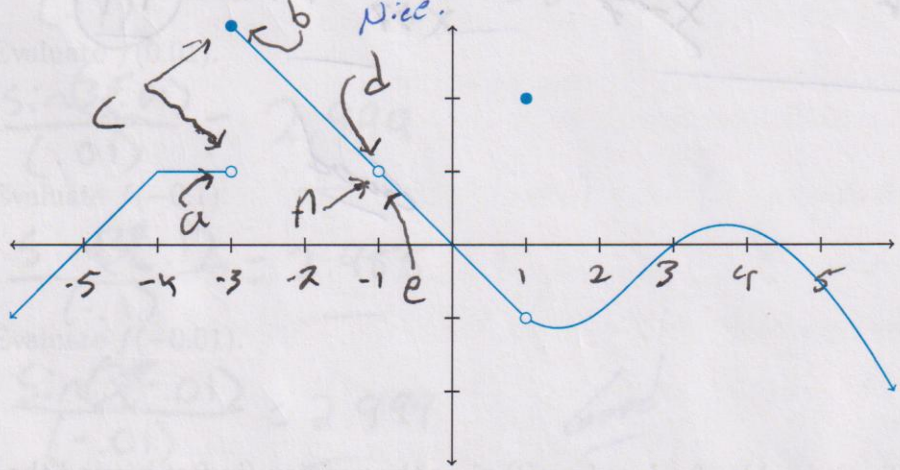
Exam 1b **Calc 1** **9/12/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:

Nice!



1. (a) What is $\lim_{x\to -3^-} f(x)$? 1

   (b) What is $\lim_{x\to -3^+} f(x)$? 3

   (c) What is $\lim_{x\to -3} f(x)$? DNE Yes.

   (d) What is $\lim_{x\to -1^-} f(x)$? 1

   (e) What is $\lim_{x\to -1^+} f(x)$? 1

   (f) What is $\lim_{x\to -1} f(x)$? 1

2. Use interval notation to indicate where the function above is continuous.

$(-\infty,-3)\cup(-3,-1)\cup(-1,1)\cup(1,\infty)$

Great

3. Evaluate $\lim_{x \to 7} \frac{x^2 - 49}{x - 7}$.

$$\lim_{x \to 7} \frac{(\cancel{x-7})(x+7)}{\cancel{x-7}} = \lim_{x \to 7} 7+7 = 14$$

Good

4. The following table of values for $p(t)$ gives the position (in feet beyond the point where the car was at time 0) of a car t seconds after the driver hits the brakes because they saw a glowing purple giraffe.

t	$p(t)$
0.0	0
0.25	14
0.5	26
0.75	37
1.0	45

(a) What is the car's average velocity over the period from $t = 0.0$ to $t = 0.5$?

$$\frac{p(0.5) - p(0)}{0.5 - 0} = \frac{26-0}{0.5-0} = \frac{26}{0.5} = \boxed{52 \text{ ft/s}}$$

Excellent

(b) What is the car's average velocity over the period from $t = 0.0$ to $t = 0.25$?

$$\frac{p(0.25) - p(0)}{0.25 - 0} = \frac{14-0}{0.25-0} = \frac{14}{0.25} = \boxed{56 \text{ ft/s}}$$

5. Let $f(x) = \dfrac{\sin 3x}{x}$. **Make sure your calculator is in radian mode.**

(a) Evaluate $f(0.1)$.

2.955

(b) Evaluate $f(0.01)$.

2.999

(c) Evaluate $f(-0.1)$.

2.955

(d) Evaluate $f(-0.01)$.

2.999

Great

(e) What's $\lim_{x \to 0} f(x)$?

3 on both sides of zero, as it gets closer to 0, the limit gets closer to 3.

6. [WeBWorK] A ball is thrown into the air by a baby alien on a planet in the system of Alpha Centauri with a velocity of 30 ft/s. Its height in feet after t seconds is given by $h(t) = 30t - 27t^2$. Find the average velocity for the time period beginning when $t_0 = 0.5$ seconds and lasting for 0.1 seconds.

$t = 0.5s$

$h(0.5) = 30(0.5) - 27(0.5)^2$

$15 - 6.75$

$h(0.5) = 8.25$ ft

$t = 0.5 + 0.1$

$t = 0.6$

$h(0.6) = 30(0.6) - 27(0.6)^2$

$18 - 9.72$

$h(0.6) = 8.28$ ft

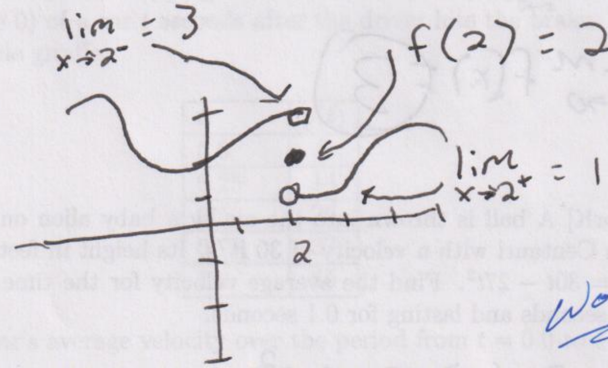
$$\frac{8.28 - 8.25}{0.6 - 0.5} = \frac{0.03}{0.1} = \boxed{0.3 \text{ ft/s}}$$

Excellent

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Whoa, Calc is kicking my butt! I can plug numbers into functions, you know? But now there's this limit stuff and it's totally confusing. Like, the professor was saying that for this function the limit when x was 2 didn't exist, and what about f of 2. I said it must not exist too, but this other person in the class said it could. Isn't that mixed up?"

   Help Biff by explaining, in terms he can understand, what can be concluded about $f(2)$ from the information given.

Although the limit doesn't exist (meaning that approaching $x=2$ from the left and the right are not at the same height) doesn't mean the value $f(2)$ doesn't exist it just means it isn't continuous. For example, a graph of this situation could look like this...



Wonderful.

8. Consider the function defined by

$$g(x) = \begin{cases} 3x^2 - 2, & x \le 1 \\ 5 - 4x, & x > 1 \end{cases}$$

Is $g(x)$ continuous at $x = 1$? Explain clearly why or why not.

$\lim_{x \to 1^-} g(x) \quad 3(1)^2 - 2 = 3 - 2 = \underline{1}$

$\lim_{x \to 1^+} g(x) \quad 5 - 4(1) = 5 - 4 = 1$

$\lim_{x \to 1} g(x) \quad 3(1)^2 - 2 = 3 - 2 = 1$

Yes, g(x) is continuous because I looked at the piece wise function and tested What is the limit when approaching from each side. Also the limit exactly at 1. All 3 of my answers were 1 meaning they come in at the same height. Making g(x) continuous.

Excellent!

9. Evaluate $\lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$. $= \lim_{h \to 0} \frac{(h^2 + 2ha + \cancel{a^2}) - \cancel{a^2}}{h}$

$= \lim_{h \to 0} \frac{h^2 + 2ha}{h} = \lim_{h \to 0} \frac{\cancel{h}(h+2a)}{\cancel{h}} = \lim_{h \to 0} h + 2a = 0 + 2a = \boxed{2a}$

Excellent!

10. Evaluate $\lim_{x \to 2} \frac{|x-2|}{|x-4| - x}$. Be sure to include clear justification.

$\lim_{x \to 2^-} \frac{|1.9-2|}{|1.9-4|-1.9} = \frac{.1}{.2}$

$\lim_{x \to 2^+} \frac{|1.99-2|}{|1.99-4|-1.99} = \frac{.01}{.02}$

$\lim_{x \to 2^+} \frac{|2.1-2|}{|2.1-4|-2.1} = \frac{-.1}{.2}$

$\lim_{x \to 2^+} \frac{|2.01-2|}{|2.01-4|-2.01} = \frac{-.01}{.02}$

$\lim_{x \to 2}$ DNE

The limits as the function approaches from the right and left do not match up making this limit not existent

Good