Exam 2c **Calc 1** **10/3/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

W

Good

2. [WeBWorK] Find an equation for the line tangent to the graph of

$$f(x) = -5xe^x$$

$(3, -15e^3)$

at the point $(a, f(a))$ for $a = 3$.

to find $f(a)$: $-5(3)e^3$
$= -15e^3$

$f'(x) = -5 \cdot e^x + -5x \cdot e^x$
$= -5e^x + (-5xe^x)$
$= -5e^x(1+x)$

plug for x $= -5e^3(1+3)$
$= -5e^3(4)$

$f'(x) = -20e^3$ (slope)

$y - y_0 = m(x - x_0)$

$\boxed{y - (-15e^3) = -20e^3(x-3)}$

Excellent!

$$\frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2}$$

3. [WeBWorK] Let $f(x) = \frac{4}{3x+6}$. Find $f'(x)$ Quotient Rule

$f(x)$, $g(x)$

$$f'(x) = \frac{0\cdot(3x+6) - 4\cdot(3+0)}{(3x+6)^2}$$

$$f'(x) = \frac{-12}{(3x+6)^2}$$

Great!

4. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x}$.

$$\frac{1}{2\sqrt{x}}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

input $= 0$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x} + \sqrt{x}}$$

Combined

$$= \frac{1}{2\sqrt{x}}$$

Excellent!

5. Show why the derivative of $\tan x$ is $\sec^2 x$.

$$(\tan x)' = \frac{\sin x}{\cos x} \to \text{Quotient rule!}$$

$$(\tan x)' = \frac{\cos x \cdot \cos x - \sin x \cdot \sin x}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

Great

$$= \boxed{\sec^2 x}$$

6. Prove the Product Rule for derivatives.

$$[f(x)\cdot g(x)]' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) \overbrace{- g(x+h)\cdot f(x) + f(x)\cdot g(x+h)}^{\text{add zero}} - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \cdot g(x+h) + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \cdot f(x)$$

because $g(x)$ and $f(x)$ are differentiable

$$= \lim_{h\to 0} f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Excellent!

7. Biff is a calculus student at Enormous State University, and he's having some trouble with derivatives. Biff says "Dude, I think calculus is broken! Our TA said that this one problem, like with the e to x thing over x squared, right? He said that instead of doing the quotient rude thing on it, you could do it by the product rude thing. Obviously that's wacked, because what I know for sure is that in math there's just one right way to do things, right?"

$e^x \cdot x^{-2}$

x^2

Help Biff by explaining, in terms he can understand, either how there can be the two different approaches his TA mentioned, or why there can't be.

Hey Biff, in Math many times you'll find you can do something more than one way. Quotient rule is absolutely right, but don't you think product rule is easier than quotient rule?. Biff: "yes".

Great! So we can actually turn this into product rule by converting the denominator x^2 into x^{-2}. The negative tells us it is on the bottom, but allows for us to do product rule when in this form. Let's visualize it!

Product	Quotient ($\frac{\text{Low} \cdot \text{DHigh} - \text{High} \cdot \text{DLow}}{\text{Sq. of below}}$
$\frac{e^x}{x^2}$	$\frac{e^x}{x^2}$
$e^x \cdot x^{-2}$	$\frac{x^2 \cdot e^x - e^x \cdot 2x}{(x^2)^2}$
$e^x \cdot x^{-2} + e^x \cdot -2x^{-3}$	

Excellent!

Both ways

8. Show why the derivative of $\ln x$ is $1/x$.

$f(x) = \ln x$

$f'(x) = ?$

we know $e^{\ln x} = x$

Chain Rule

$$\frac{e^{\ln x} \cdot (\ln x)' = 1}{e^{\ln x}}$$

$(\ln x)' = \frac{1}{e^{\ln x}} \rightarrow (\ln x)' = \frac{1}{x}$

Great

SOH CAH TOA

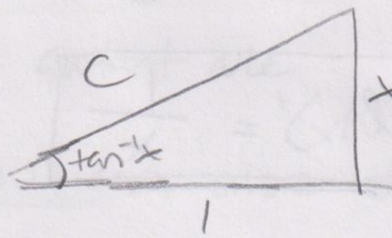
9. Show why the derivative of $\tan^{-1} x$ is $\dfrac{1}{1+x^2}$.

$$\tan(\tan^{-1}x) = x$$

$$\sec^2(\tan^{-1}x)\cdot(\tan^{-1}x)' = 1$$

$$(\tan^{-1}x)' = \frac{1}{\sec^2(\tan^{-1}x)}$$

$$= \frac{1}{\frac{1}{\cos^2(\tan^{-1}x)}}$$



$$1^2 + x^2 = c^2$$

$$\sqrt{c^2} = \sqrt{1+x^2}$$

$$c = \sqrt{1+x^2}$$

$$= \cos^2(\tan^{-1}x)$$

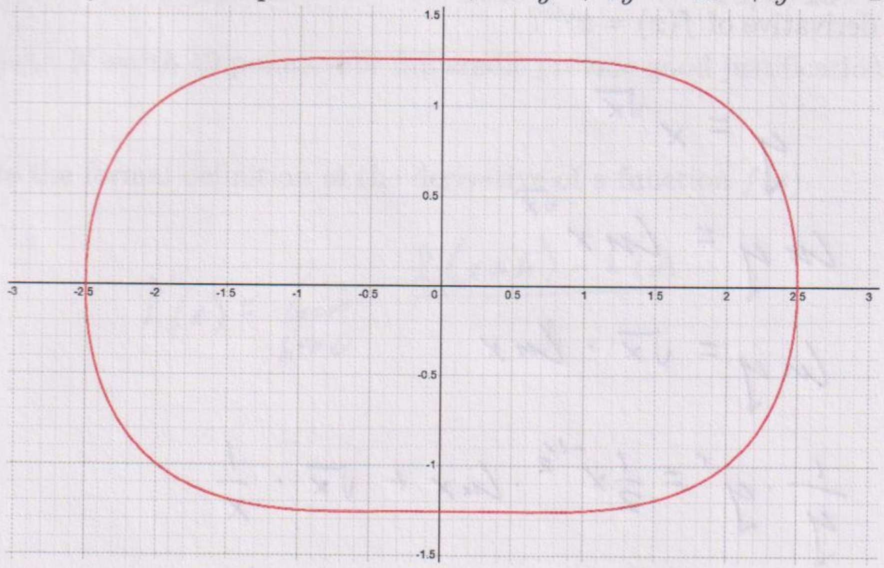
$$= [\cos(\tan^{-1}x)]^2$$

$$= \left(\frac{1}{\sqrt{1+x^2}}\right)^2$$

$$= \boxed{\frac{1}{1+x^2}}$$

Excellent!

10. (a) Find y' for the implicit curve $x^4 + 2x^2y^2 + 5y^4 - 4x^2 + y^2 \ = \ 14$.



$$4x^3 + 4x \cdot y^2 + 2x^2 \cdot 2y \cdot y' + 20y^3 \cdot y' - 8x + 2y \cdot y' = 0$$

$$4x^2 y \cdot y' + 20y^3 \cdot y' + 2y \cdot y' = 8x - 4x^3 - 4xy^2$$

$$y'(4x^2y + 20y^3 + 2y) = 8x - 4x^3 - 4xy^2$$

$$y' = \frac{8x - 4x^3 - 4xy^2}{4x^2y + 20y^3 + 2y}$$

(b) What is the slope of the tangent line to the curve from part a at the point $(2, -1)$?

$$y' = \frac{8(2) - 4(2)^3 - 4(2)(-1)^2}{4(2)^2(-1) + 20(-1)^3 + 2(-1)}$$

$$= \frac{16 - 32 - 8}{-16 - 20 - 2}$$

$$= \frac{-24}{-38} = \frac{12}{19}$$