Exam 3a **Calc 1** **10/30/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Use interval notation to express where $f(x) = x^3 - 3x^2 + 5$ is increasing.

$f(x) = x^3 - 3x^2 + 5$

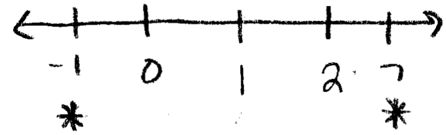
$f'(x) = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0 \quad x = 2$

CRITICAL POINTS!



TESTS

$f'(-1) = 3(-1)^2 - 6(-1) = 9$ ⊕

$f'(1) = 3(1)^2 - 6(1) = -3$ ⊖

$f'(7) = 3(7)^2 - 6(7) = 105$ ⊕

$(-\infty, 0) \cup (2, \infty)$

Excellent!

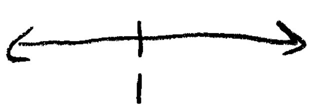
2. Use interval notation to express where $f(x) = x^3 - 3x^2 + 5$ is concave up.

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

$0 = 6(x-1)$

$x = 1$



$f''(0) = -6$

$f''(2) = 6$ concave up

$\boxed{(1, \infty)}$

Excellent!

$f(x) = x^3 - 3x^2 + 5$ is concave up on the interval $(1, \infty)$

3. Let $f(x) = x^2 - x$. Find the absolute minimum and absolute maximum values of f on the interval $[0, 3]$.

$f'(x) = 2x - 1$

$0 = 2x - 1$

$\frac{1}{2} = x \rightarrow$ critical points

Test

$f(0) = (0)^2 - 0 = 0$ min

$f(\frac{1}{2}) = (\frac{1}{2})^2 - (\frac{1}{2}) = -\frac{1}{4}$

$f(3) = (3)^2 - 3 = 6$

absolute minimum is at $(-\frac{1}{4})$ & absolute maximum is at 6

Excellent

4. The profit function for a computer company is given by $P(x) = -x^2 + 28x - 26$ where x is the number of units produced (in thousands) and the profit is in thousand of dollars. Determine how many (thousands of) units must be produced to yield maximum profit.

$$P'(x) = -2x + 28$$

$$0 = -2x + 28$$

$$-28 = -2x$$

$$x = 14$$

14 units (thousands of) must be produced to yeild max profit (which would make

170,000

Excellent!

5. Use Newton's Method with the function $f(x) = x^3 - 5$ and initial value $x_0 = 1.6$ to calculate x_1.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x)$

$f(x) = 3x^2$

$$x_1 = x_0 = \frac{f(x_0)}{f'(x_0)}$$

Great!

$f(1.6) = (1.6)^3 - 5 = 4.096 - 5 = -0.904$

$f'(1.6) = 3(1.6)^2 = 3(2.56) = 7.68$

$$x_1 = 1.6 - \frac{-0.904}{7.68} = 1.6 + \frac{0.904}{7.68} = 1.6 + 0.117708333 \approx 1.7177$$

$x_1 = 1.718$

6. Evaluate $\lim_{x\to\infty} \frac{\ln x}{1 + (\ln x)^2}$.

$$\overset{\text{LH}}{=} \lim_{x\to\infty} \frac{\cancel{\frac{1}{x}}}{2(\ln x) \cdot \cancel{\frac{1}{x}}} = \lim_{x\to\infty} \frac{1}{2 \ln x} = 0$$

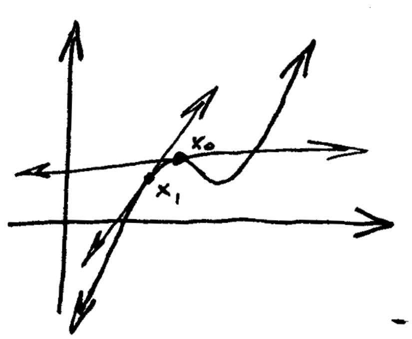
∞ ∞

Excellent!

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so confusing? I can work out *answers*, you know, but now they ask these questions that aren't just about getting answers. So like, with the Fig Newton's Method thingy, there was this question on our test about if you got dividing by zero did that mean there was no solution. Since when is 'no solution' an answer?"

Help Bunny by explaining as clearly as you can what it means when dividing by zero comes up in using Newton's Method, and whether it always means there's no solution.

Hey Bunny, don't panic. You just need to understand how Newton's Method (It's Isaac Newton, b.t.w., not Fig) works. It uses tangent lines to approximate places where a function crosses the x-axis. So like in this picture, the

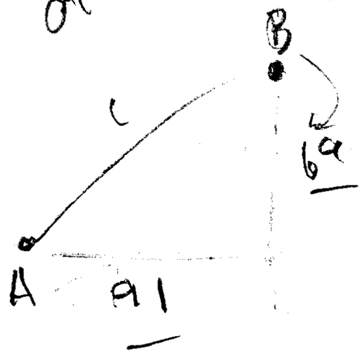


tangent line at x_1 crosses the x-axis pretty close to where the function crosses the axis, right? So Newton's Method finds that place. But it can depend a lot on your starting point. If you use the tangent line at x_0, it's horizontal, right? So it never hits the x-axis at all. In the formula that corresponds to dividing by 0 because the denominator, $f'(x_0)$, is 0 there.

So all of that doesn't mean the actual function doesn't have a solution. It might just be about a bad first guess, like with x_0 in that picture. Try a different starting point, or maybe look at a graph to get a better idea of where to start or what's going on.

8. [WW]At noon, ship A is 40 nautical miles due west of ship B. Ship A is sailing west at 17 knots and ship B is sailing north at 23 knots. How fast (in knots) is the distance between the ships changing at 3 PM?

$\frac{dA}{dt} = 17$, $\frac{dB}{dt} = 23$



~~Ass~~

At 3Pm ship A has gone 51 nm and ship B has gone 69 nm

Great

$A^2 + B^2 = C^2 \longrightarrow 91^2 + 69^2 = C^2$

$C = \sqrt{13042}$

≈ 114.2

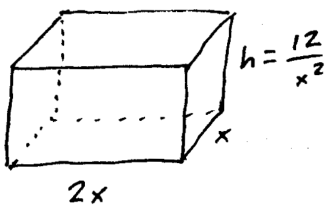
$2A \cdot \frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dc}{dt}$

$2(91) \cdot 17 + 2(69) \cdot 23 = 2(\sqrt{13042}) \cdot \frac{dc}{dt}$

$\frac{dc}{dt} = \frac{3094 + 3174}{2\sqrt{13042}}$

$\frac{dc}{dt} \approx$ 27.443 knots ~~per hour~~

9. [WW]A rectangular storage container with an open top is to have a volume of 24 cubic feet. The length of its base is twice the width. Material for the base costs 14 dollars per square foot. Material for the sides costs 5 dollars per square foot. Find the cost of materials for the cheapest such container.



$$(2x)(x)(h) = 24$$

$$h = \frac{24}{2x^2} = \frac{12}{x^2}$$

$$C(x) = (2x)(x)(14) + (2x)\left(\frac{12}{x^2}\right)(5) + (x)\left(\frac{12}{x^2}\right)(5)$$

$$C(x) = 28x^2 + 120x^{-1} + 60x^{-1}$$

$$C(x) = 28x^2 + 180x^{-1}$$

I. $C'(x) = 56x - 180x^{-2}$

II. $0 = 56x - 180x^{-2}$

$$0 = 56x^3 - 180$$

$$0 = 28x^3 - 90$$

$$90 = 28x^3$$

$$x^3 = \frac{90}{28}$$

$$x = \sqrt[3]{\frac{90}{28}} \approx 1.458$$

$$C\left(\sqrt[3]{\frac{90}{28}}\right) \approx \$182.95$$

$$28x^2 + 240x^{-1} + 120x^{-1}$$

$$C(x) = 28x^2 + 360x^{-1}$$

$$C'(x) = 56x - 360x^{-2}$$

$$0 = 56x - \frac{360}{x^2}$$

$$0 = 56x^3 - 360$$

$$360 = 56x^3$$

$$x^3 = \frac{360}{56}$$

$$x = \sqrt[3]{\frac{45}{7}} \approx 1.86$$

$$C(1.86) \approx \$290.42$$

10. Jon is planning to start a Youtube CalculusCheats channel and monetize it heavily. He has sponsors lined up to pay for advertisements. Jon projects 45 thousand views if he runs no advertisements (apart from the ones Youtube includes, since that revenue doesn't go to Jon) but expects to lose 5 thousand views for each minute of paid advertising that he includes. If he's paid \$0.10 for each minute of ads that each viewer watches, how many minutes of ads should he include to maximize his revenue?

$R = 0.10x(45000 - 5000x)$

$R' = 0.1 \cdot (45000 - 5000x) + 0.1x \cdot -5000$

$= 4500 - 500x - 500x$

$= 4500 - 1000x$

$0 = 4500 - 1000x$

$1000x = 4500$

$x = 4.5$

Great!

He should run 4.5 minuts of ads to maximize his revenue.