Exam 3b **Calc 1** **10/31/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formula for Newton's Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2. Use interval notation to express where $f(x) = x^3 - 6x + 5$ is increasing.

$f'(x) = 3x^2 - 6$

$0 = 3(x^2 - 2)$

$0 = 3(x + \sqrt{2})(x - \sqrt{2})$

$x = -\sqrt{2}$ or $x = \sqrt{2}$

Test:

$f'(-3) = +$

$f'(0) = -$

$f'(3) = +$

So it's increasing where f' is positive,

$(-\infty, -\sqrt{2} \cup (\sqrt{2}, \infty)$

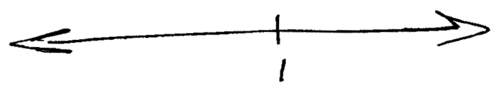
3. Use interval notation to express where $f(x) = x^3 - 3x^2 + 5$ is concave up.

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

$f'(x) = 6(x-1)$

$x = 1$



Test:

$f''(0) = -$

$f''(2) = +$

So it's concave up where f'' is positive,

$(1, \infty)$

4. Let $f(x) = 2x^3 - 15x^2 + 24x + 1$. Find the absolute minimum and absolute maximum values of f on the interval $[0, 3]$.

$f'(x) = 6x^2 - 30x + 24$

$f'(x) = 6(x^2 - 5x + 4) = 6(x-4)(x-1)$

$x = 4$ or $x = 1$

Test endpoints and any critical points within the interval:

$f(0) = 1$

$f(1) = 2(1)^3 - 15(1)^2 + 24(1) + 1 = 2 - 15 + 24 + 1 = 12$

$f(3) = 2(3)^3 - 15(3)^2 + 24(3) + 1$

$= 54 - 135 + 72 + 1$

$= -8$

abs. min. value

abs. max value

5. Use Newton's Method with the function $f(x) = x^3 - 5$ and initial value $x_0 = 1.6$ to calculate x_1.

$$f'(x) = 3x^2$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$x_1 = 1.6 - \frac{f(1.6)}{f'(1.6)} = 1.6 - \frac{-0.904}{7.68}$$

$$\approx 1.717708333$$

6. Evaluate $\displaystyle\lim_{x \to 0^+} \frac{\ln x}{1 + (\ln x)^2}$.

It's an $\frac{\infty}{\infty}$ indeterminate form, so

$$\overset{L'H}{=} \lim_{x \to 0^+} \frac{1/x}{2(\ln x) \cdot \frac{1}{x}}$$

$$= \lim_{x \to 0^+} \frac{1}{2 \ln x}$$

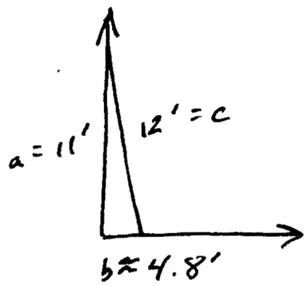
$$= 0$$

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so unfair? I really like the Hospital Rule thingy, but then the test question was totally messed up! It was like cos over $x - 1$, and I did the derivatives just like you're supposed to, but they said I got the wrong answer and got no partial credit at all because I was invalid. How can they call me invalid?"

Help Bunny by explaining as clearly as you can why her approach might not have been a good choice for a problem like this.

Bunny the L'Hopital rule only occurs when the fraction comes out to be $\frac{0}{0}$ or $\frac{\infty}{\infty}$. When this happens you use L'Hop's rule and do the derivative of the top and the derivative of the bottom until the fraction is not $\frac{0}{0}$ or $\frac{\infty}{\infty}$. $\frac{\cos}{x-1}$ isn't $\frac{0}{0}$ so you can't apply L'Hop's rule.

8. [WW]A 12 foot ladder is leaning against a wall. If the top slips down the wall at a rate of 3 ft/s, how fast will the foot be moving away from the wall when the top is 11 feet above the ground?



$$a^2 + b^2 = c^2$$
$$11^2 + b^2 = 12^2$$
$$b^2 = 144 - 121$$
$$b^2 = 23$$
$$b \approx 4.79583$$

So since $a^2 + b^2 = c^2$, differentiating gives

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(11)(-3) + 2(4.8)\left(\frac{db}{dt}\right) = 2(12)^2\cdot(0)$$

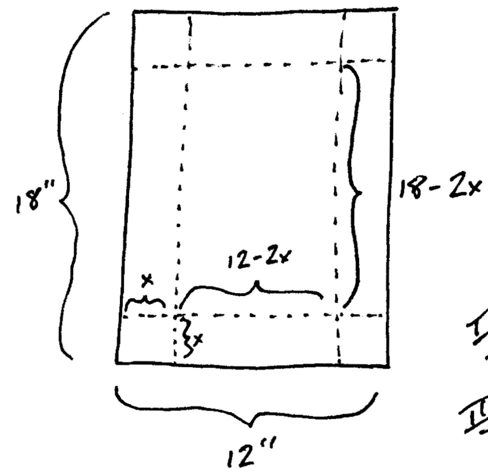
$$-66 + 9.6\frac{db}{dt} = 0$$

$$9.6\frac{db}{dt} = 66$$

$$\frac{db}{dt} = \frac{66}{9.6}$$

$$\frac{db}{dt} \approx 6.880975664 \text{ ft/s}$$

9. [WW]A box is to be made out of a 12 in by 18 in piece of cardboard. Squares of side length x in will be cut out of each corner, and then the ends and sides will be folded up to form a box with an open top. Find the maximum volume possible for the box.



$$V(x) = (18-2x)(12-2x)x$$
$$= (216-36x-24x+4x^2)x$$
$$V(x) = 216x - 60x^2 + 4x^3$$

I. Diff: $V'(x) = 216 - 120x + 12x^2$

II. set=0 $0 = 12(18 - 10x + x^2)$

That doesn't factor nicely, so we'll use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(18)}}{2(1)}$$
$$= \frac{10 \pm \sqrt{100-72}}{2}$$
$$= \frac{10 \pm \sqrt{28}}{2}$$
$$= \frac{10 \pm 2\sqrt{7}}{2}$$
$$= 5 \pm \sqrt{7}$$

So $5+\sqrt{7}$ is crazy big and can't be a minimum, so we want

$x = 5-\sqrt{7} \approx 2.354$

Then $V(5-\sqrt{7}) \approx 228.162$ in^2

10. A smokestack deposits soot on the ground with a concentration inversely proportional to the square of the distance from the stack. With two smokestacks d miles apart, the concentration of the combined deposits on the line joining them, at a distance x from one stack, is given by

$$S = \frac{c}{x^2} + \frac{k}{(d-x)^2}$$

where c and k are positive constants which depend on the quantity of smoke each stack is emitting. If $k = 4c$, find the point on the line joining the stacks where the concentration of the deposit is a minimum.

$$S(x) = \frac{c}{x^2} + \frac{4c}{(d-x)^2} = cx^{-2} + 4c(d-x)^{-2}$$

I. $S'(x) = -2cx^{-3} + -8c(d-x)^{-3} \cdot -1$

$$0 = \frac{-2c}{x^3} + \frac{8c}{(d-x)^3}$$

$$0 = \frac{4}{(d-x)^3} - \frac{1}{x^3}$$

$$0 = 4x^3 - (d-x)^3$$

$$0 = 4x^3 - (d^3 - 3d^2x + 3dx^2 - x^3)$$

$$0 = 5x^3 - 3dx^2 + 3d^2x - d^3$$

$(d-x)(d-x)(d-x)$

$(d^2 - 2dx + x^2)(d-x)$

$d^3 - d^2x - 2d^2x + 2dx^2$

$+ dx^2 - x^3$

$$(d-x)^3 = 4x^3$$

$$d - x = x\sqrt[3]{4}$$

$$d = x + x\sqrt[3]{4}$$

$$d = x(1 + \sqrt[3]{4})$$

$$x = \frac{d}{1 + \sqrt[3]{4}}$$