Exam 4 **Calc 1** **11/15/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Based on the values given in the table,

x	1	2	3	4	5
$f(x)$	-5	-1	-3	1	16
$f'(x)$	9	0	-3	0	9
$f''(x)$	-12	-6	0	6	12

(a) What are the critical numbers of f?

$x=2$ $x=4$

where the first derivative of $f(x)$ is equal to zero.

2. Find the interval(s) on which $f(x) = x^3 - 9x^2 + 4$ is decreasing.

$f'(x) = 3x^2 - 18x$

$3x^2 - 18x = 0$

$3x(x-6) = 0$ $x=6$ or $x=0$

$f'(-1) = +$

$f'(1) = -$

$f'(7) = +$

$(-\infty, 0)$ $(0, 6)$ $(6, \infty)$

$+$ $-$ $+$

$f(x)$ is decreasing on the interval $(0, 6)$

Great

3. Find the interval(s) where $f(x) = x^3 - 9x^2 + 4$ is concave up.

$f'(x) = 3x^2 - 18x$

$f''(x) = 6x - 18$

$6x - 18 = 0$

$6x = 18$

$x = 3$

$f''(0) = -$

$f''(4) = +$

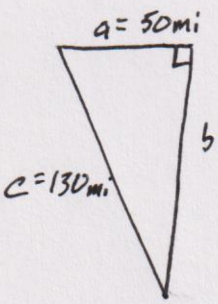
$(-\infty, 3)\ (3, \infty)$

$-\qquad +$

Great!

$f(x)$ is concave up on the interval $(3, \infty)$

4. [Stewart]Two cars start moving from the same point. One travels $\underbrace{\text{south at 60mi/h}}_{\frac{db}{dt} = 60}$ and the other travels $\underbrace{\text{west at 25 mi/h}}_{\frac{da}{dt} = 25}$. At what rate is the distance between the cars increasing two hours later?



$a^2 + b^2 = c^2$

$(50)^2 + (120)^2 = c^2$

$16900 = c^2$

$c = 130mi$

I know:

$a^2 + b^2 = c^2$

so $2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(50)(25) + 2(120)(60) = 2(130)\frac{dc}{dt}$

$16900 = 260\frac{dc}{dt}$

$\frac{dc}{dt} = \frac{16900}{260}$

$\frac{dc}{dt} = 65 mi/h$

5. Find the absolute maximum and absolute minimum values of $f(x) = xe^{-x}$ on $[0, 3]$

I. $f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot -1 = e^{-x} - xe^{-x}$

II. $0 = e^{-x}(1-x)$

$x = 1$

So check critical points and endpoints:

$f(0) = 0 \cdot e^{-0} = 0$ smallest, so min

$f(1) = 1 \cdot e^{-1} = 1/e \approx 0.368$ biggest, so max

$f(3) = 3 \cdot e^{-3} = 3/e^3 \approx 0.149$

6. Two real numbers add up to 25. What is the largest their **product** can be?

$x + y = 25$ $\quad y = 25 - x$

$f(x) = x \cdot y = x(25 - x)$

$f(x) = 25x - x^2$

$f'(x) = 25 - 2x$

Excellent!

$25 - 2x = 0$ $\quad 2x = 25$ $\quad x = 12.5$

$y = 25 - 12.5$

$y = 12.5$

$12.5 \cdot 12.5 = 156.25$

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so confusing? I get the slopey parts, you know? But then they have this cavity part, which makes no sense because that's teeth, right? But so somehow the cavity tells you a max instead of a min or something, right? What's up with that?"

Help Bunny by explaining as clearly as you can how concavity connects to maxes and mins.

Bunny concavity isn't teeth related. ☺

It is were you find the second derivative of a function and determine what shape of the graph it is. You create intervals with x values from setting the second derivate to zero, don't forget infinity plays a huge role bunny. Once you do this you create intervals and plug in a number that fits in that interval in the second derivative. If it comes out positive you have a concave up, negative concave down. If it is a concave up that x-value in a minimum, if concave down x-value is a maximum. For example,

Excellent!

$f(x)\quad 3x^3-2x^2$

$f(x)=9x^2-4x$

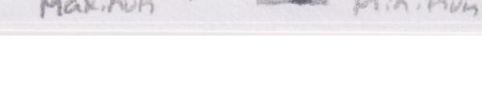
$f''(x)=18x-4$

$0=18x-4$

$4/18=x$

plug in 0 — $(-\infty, 4/18)$ — $\ominus$ — concave down

plug in 1 — $(4/18, \infty)$ — $\oplus$ — concave up



8. Approximate $\sqrt[3]{2}$ using Newton's Method with an initial value $x_0 = 2$ to calculate x_1 and x_2.

$\sqrt[3]{2}$ is a solution to $x^3 = 2$, or $0 = x^3 - 2$,

So we use Newton's Method on $f(x) = x^3 - 2$.

Then $f'(x) = 3x^2$, and

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = (2) - \frac{f(2)}{f'(2)} = 2 - \frac{6}{12} = \frac{3}{2}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{3}{2} - \frac{f(3/2)}{f'(3/2)} = \frac{3}{2} - \frac{\frac{27}{8} - 2}{3 \cdot 9/4}$$

$$= \frac{3}{2} - \frac{11/8}{27/4} = \frac{3}{2} - \frac{11}{8} \cdot \frac{4}{27} = \frac{3}{2} - \frac{11}{54}$$

$$= \frac{81}{54} - \frac{11}{54} = \frac{70}{54} = \frac{35}{27} \approx 1.296296$$

9. Jon is planning to start selling defective airpods on Temu. His research shows that if he sells them for \$20 he'll sell 500 per month, whereas if he sells them for \$21 he'll sell 450 per month. Assuming that demand is linear, what **price** should he set to maximize his revenue?

$f(x) = (20 + x)(500 - 50x)$

$f(x) = 10000 - 1000x + 500x - 50x^2$

$f(x) = 10000 - 500x - 50x^2$

$f'(x) = -500 - 100x$

$-500 - 100x = 0$

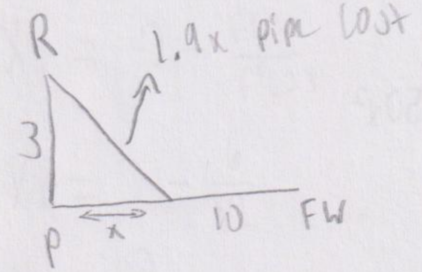
$100x = -500$

$x = -5$

To maximize his revenue, Jon should start selling his defective airpods at \$15.

Excellent.

10. [WW] A small resort is situated on an island that lies exactly 3 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 1.9 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?



$f(x) = 1.9\sqrt{3^2+x^2} + (10-x)$

The pipe should reach land $\sqrt{\frac{9}{2.61}}$ miles, or 1.8569533 miles, down the shoreline from P.

Excellent!

$f'(x) = \frac{1.9}{2\sqrt{3^2+x^2}} \cdot 2x - 1$

$\frac{1.9x}{\sqrt{3^2+x^2}} - 1 = 0$

$\frac{1.9x}{\sqrt{3^2+x^2}} = 1$

$1.9x = \sqrt{3^2+x^2}$

$3.61x^2 = 3^2 + x^2$

$2.61x^2 = 9$

$x^2 = \frac{9}{2.61}$

$x = \sqrt{\frac{9}{2.61}}$

$x \approx 1.8569533$