Exam 4a **Calc 1** **11/21/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the definition of the definite integral of a function $f(x)$ on the interval $[a, b]$.

$$\int_b^a f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

Good

10

2. Evaluate $\int_0^9 \sqrt{x}\, dx$ exactly. $= \int_0^9 x^{1/2} dx$

$$= \frac{2}{3} x^{3/2} \Big|_0^9$$

$$= \frac{2}{3}(9)^{3/2} - \frac{2}{3}(0)^{3/2}$$

$$= \boxed{18}$$

Good

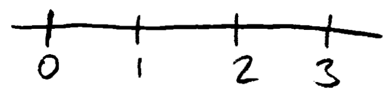
10

3. The following table of values for $v(t)$ gives the position (in feet beyond the point where the car was at time 0) of a car t seconds after the driver hits the brakes because they saw a glowing purple giraffe.

t	$v(t)$
0.0	88
0.5	78
1.0	56
1.5	35
2.0	20
2.5	9
3.0	0

Find L_3, R_3, and M_3, the left-hand, right-hand, and midpoint approximations for the distance travelled by the car over the three seconds it takes to come to a stop.

$$L_3 = f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1$$
$$= 88 + 56 + 20$$
$$\boxed{L_3 = 164 \text{ ft}}$$



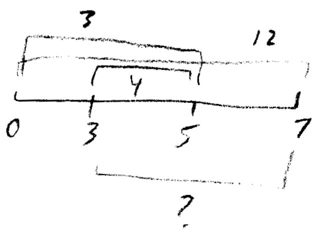
$$R_3 = f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$$
$$= 56 + 20 + 0$$
$$\boxed{R_3 = 76 \text{ ft}}$$

Great

$$M_3 = f(0.5)\cdot 1 + f(1.5)\cdot 1 + f(2.5)\cdot 1$$
$$= 78 + 35 + 9$$
$$\boxed{M_3 = 122 \text{ ft}}$$

4. If $\int_0^7 f(x)\,dx = 12$, $\int_0^5 f(x)\,dx = 3$, and $\int_3^5 f(x)\,dx = 4$, what is $\int_3^7 3 \cdot f(x)\,dx$ and why?

$$\int_0^7 f(x)\,dx - \int_0^5 f(x)\,dx + \int_3^5 f(x)\,dx$$



$12 - 3 + 4 = \underline{13}$

$15 + 4$

$$\begin{array}{r} 13 \\ \times\ 3 \\ \hline 39 \end{array}$$

Excellent!

You need to remove $\int_0^5 f(x)dx$ to find $\int_5^7 f(x)dx$. From there, you can add $\int_3^5 f(x)\,dx$ to find $\int_3^7 f(x)$. This just needs to be multiplied by 3 to find $\int_3^7 3\cdot f(x)\,dx$

$$\int_3^7 3\cdot f(x)\,dx = \underline{39}$$

5. A ball is thrown off a 100 ft cliff with an initial upward velocity of 96 ft/s. The acceleration due to gravity is -32 ft/s^2. Find the ball's height above the bottom of the cliff after 5 seconds.

$a(t) = -32$

$v(t) = -32t + 96$

$p(t) = \underline{-16t^2 + 96t + 100}$

the "+C" is 96 bc $v(0) = 96$

the "+C" is 100 bc $p(0) = 100$

$p(5) = \underline{-16(5)^2 + 96(5) + 100}$

$= \underline{180}$ ft above the bottom of the cliff

Great

6. If a company's expected profits t years from now, in millions of dollars per year, are given by $p(t) = 40 + 5t$, what total profit can the company expect over the next five years?

$$\int_0^5 (40 + 5t)\,dt$$

$$\int (40 + 5t)\,dt = 40t + \frac{5}{2}t^2 + C$$

$$\left[40t + \frac{5}{2}t^2\right]_0^5 = \left(40(5) + \frac{5}{2}(5)^2\right) - \left(40(0) + \frac{5}{2}(0)^2\right)$$

$$= \left(200 + \frac{5}{2}(25)\right) - 0$$

Great

$$= 200 + \frac{125}{2}$$

$$= 200 + 62.5$$

$$= 262.5$$

262.5 million dollars

7. The first edition of the *Cliff'sNotes Calculus Quick Review* says on page 116 that the area A of the region bounded by the graph of $f(x)$, the x axis, and the lines $x = a$ and $x = b$ is given by $A = \left| \int_a^b f(x)\,dx \right|$ when the function $f(x)$ is sometimes above and sometimes below the x axis.

   Explain in at least a few sentences why the absolute value bars in this expression do or don't accurately express the actual area bounded by these curves.

The absolute value does not accurately express the actual area bounded by these curves since if the curve is both above and below the x-axis, then the integral will cancel out and the absolute value of what you get will be less than the actual area.

Good

8. Let $f(x) = \int_0^x \cos(t^2)\,dt$.

(a) What is $f(0)$?

$$\int_0^0 \cos(t^2)$$

A integral from 0 to 0 is a line. A line does not have an area/integral to find.

$f(0) = \underline{0}$

Excellent!

(b) What is $f'(x)$?

Undo!

$$\frac{d}{dx}\int_0^x \cos(t^2)\,dt$$

Via the first part of fund. Thereom of Calculus.

$= \underline{\cos(x^2)}$

9. **Evaluate** $\int \frac{3}{(2x-8)^4}\,dx.$ $= \int \frac{3}{(u)^4} \cdot \frac{du}{2}$

Let $u = 2x-8$

$\frac{du}{dx} = 2$

$du = 2dx$

$\frac{du}{2} = dx$

$= \frac{1}{2} \int 3u^{-4}\,du$

$= \frac{1}{2} \cdot -1\, u^{-3}\,du$

$= \boxed{-\frac{1}{2}(2x-8)^{-3} + C}$

Excellent!

10. In Calc 2 you learn why $\int_0^r 2\pi x\left(h - \frac{hx}{r}\right)\,dx$ represents the volume of a cone with height h and base radius r. Evaluate this integral to find the volume. [Hint: you might want to do some algebra to simplify the function inside the integral before you try to find an antiderivative.]

$$\text{Volume} = 2\pi h \int_0^r \left(x - \frac{1}{r}x^2\right)dx$$

$$= 2\pi h\left[\frac{x^2}{2} - \frac{x^3}{3r}\right]_0^r$$

$$= 2\pi h\left[\left(\frac{r^2}{2} - \frac{r^3}{3r}\right) - \left(\frac{0^2}{2} - \frac{0^3}{3r}\right)\right]$$

$$= 2\pi h\left(\frac{3r^2}{6} - \frac{2r^2}{6}\right)$$

$$= 2\pi h\ \frac{r^2}{6}$$

$$= \frac{\pi r^2 h}{3}$$