Exam 4 Sample   **Calc 1**   **11/14/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the definition of the definite integral of a function $f(x)$ on the interval $[a, b]$.

2. Evaluate $\displaystyle\int_1^4 \sqrt{x}\, dx$ exactly.

3. The following table of values for $v(t)$ gives the position (in feet beyond the point where the car was at time 0) of a car t seconds after the driver hits the brakes because they saw a glowing purple giraffe.

t	$v(t)$
0.0	88
0.5	78
1.0	56
1.5	37
2.0	19
2.5	7
3.0	0

Find L_3, R_3, and M_3, the left-hand, right-hand, and midpoint approximations for the distance travelled by the car over the three seconds it takes to come to a stop.

4. If $\int_0^7 f(x)\,dx = 12$, $\int_0^5 f(x)\,dx = 3$, and $\int_3^5 f(x)\,dx = 4$, what is $\int_5^7 f(x)\,dx$ and why?

5. A ball is thrown into the air by a baby alien on a planet in the system of Alpha Centauri with an initial velocity of 30 ft/s. The acceleration due to gravity there is -54 ft/s^2. Find the ball's height after 1 second.

6. Suppose that $f''(x) = 3\sin x$, $f'(0) = 0$, and $f(0) = 3$. Find $f(x)$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This Calculus stuff is pretty rough. So, like, sometimes I get negative numbers when I do the definite integrate things, right? So the answer in the back of the book is pretty much always just what I got but with the negative taken off, right? So I heard it's like that because you sometimes get things upside down, like with the bottom thing first or whatever, right? So do you always just take the negative sign off?"

Help Biff by explaining as clearly as you can whether his reasoning holds, or if there are limitations.

8. Let $f(x) = \int_{\pi/2}^{x} \cos(t^2)\, dt$. What is $f'(x)$?

9. Evaluate $\int \sqrt{4x + 3}\, dx$.

10. Let a be some positive constant. Find the area of the region under the graph of $y = x(a - x)$ and above the x-axis.

Extra Credit (5 points possible):

Pick two points (let one have $x = a$ and the other have $x = b$) on the first-quadrant portion of the hyperbola $y = 1/x$. Find the area bounded by the line between those two points and the hyperbola.