Exam 1 **Calc 3** **9/27/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Show that the function $f(x, y) = \dfrac{xy + y^2}{x^2 + y^2}$ fails to have a limit at $(0, 0)$.

3. Suppose that $u = f(x, y, z)$, where $x = x(t), y = y(t)$, and $z = z(t)$. Write the Chain Rule formula for $\frac{du}{dt}$. Make very clear which derivatives are partials.

4. Find the directional derivative of $f(x, y) = 7x - x^3y + y^3$ at the point $(-1, 2)$ in the direction of the vector $\langle -3, 4 \rangle$.

5. Let $f(x, y) = \dfrac{x}{3x + 2y}$.

(a) In which direction is the directional derivative greatest at the point $(-1, 3)$?

(b) What is the value of the directional derivative in the direction of the vector from part (a)?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$ the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude! This Calc stuff is totally not even math! The professor asked us, like, what the gradient means. Like, is this philosophy or something? It means you do the f-ing partials, right? Its direction is what the f-ing partials say! It doesn't mean anything, it's just math!"

   Explain clearly to Biff what the direction of the gradient means, and how you know that.

8. Find the maximum and minimum **values** of $f(x, y) = 4x+y$ on the ellipse $x^2+49y^2 = 1$.

9. Find and classify all extrema of the function $f(x, y) = \frac{x^2}{2} + 2y^3 + 3y^2 - xy - x$ and classify each as a maximum, minimum, or saddle point.

10. Suppose you're standing at the point $(-1, 3)$ on the surface $f(x, y) = \dfrac{x}{3x + 2y}$. In some directions you face, you'll be looking uphill. In other directions, you'll be facing downhill. In which directions are you facing uphill?

Extra Credit (5 points possible): For a triangle like the one pictured below, prove the Law of Cosines, which states that $c^2 = a^2 + b^2 - 2ab\cos\gamma$.