Exam 3 **Calc 3** **11/21/25**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Give a parametrization and bounds for t to produce a counterclockwise circle with radius 5 centered at the origin.

2. Is the vector field $\mathbf{F}(x, y) = \langle xy^2, x + y^2 \rangle$ conservative? How can you be sure?

3. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = 5x^2\mathbf{i} + 8x\mathbf{j}$ and C is a path composed of a line segment from $(0, 0)$ to $(3, 0)$ followed by a line segment from $(3, 0)$ to $(0, 1)$ and then a line segment from $(0, 1)$ back to $(0, 0)$.

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + 3x^2y^2\mathbf{j}$ and C is a line segment from $(1, 1)$ to $(3, 2)$.

5. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + x^3z^5\mathbf{j} + 3z\mathbf{k}$. Let S be the sphere with radius 3, centered at the origin and oriented outward. Find $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

6. Show that for any vector field $\mathbf{F}(x, y, z)$ whose component functions have continuous partial derivatives, $\text{div}(\text{curl } \mathbf{F}) = 0$. Make it clear how the requirement that the partials be continuous is important.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This line integral stuff is crazy. There's all these different ways, but I figure the shortest is best, right? So they said something about a time it's really simple, like, if the curl of the vector field is zero then line integrals on closed paths are always zero too. Is that true?"

   Help Biff by explaining as clearly as you can whether what he heard is true, and how you know.

8. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = \langle -y, x, 5\rangle$. Let S be the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 5$. Find $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = \langle xz, y^3, -x^3 \rangle$. Let S be the portion of the paraboloid $z = x^2 + y^2$ below $z = 4$ with upward orientation. Find $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y) = \langle -y, x \rangle$, and let C be the first quadrant portion of a circle with some radius r centered at the origin, traversed counterclockwise from $(r, 0)$ to $(0, r)$. For what radius r will $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = 5$?

Extra Credit (5 points possible): If $\mathbf{a}$ is a constant vector, $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and S is an oriented, smooth surface with simple, closed, smooth, positively oriented boundary curve C, show that

$$\iint_S 2\mathbf{a} \cdot d\mathbf{S} = \int_C (\mathbf{a} \times \mathbf{r}) \cdot d\mathbf{r}$$