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Problem Set #2

① a.) Find the 7^{th} degree MacLaurin polynomial for the function $f(x) = \ln(1+x)$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \underline{= \text{in this case}} \quad f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{VII}(0)}{7!}x^7$$

$f(x) = \ln(1+x)$ | $f^{IV}(x) = -(3!)(1+x)^{-4}$ | $f(0) = 0$ | $f^{IV}(0) = -3!$

$f'(x) = (1+x)^{-1}$ | $f^{V}(x) = 4!(1+x)^{-5}$ | $f'(0) = 1$ | $f^{V}(0) = 4!$

$f''(x) = -(1+x)^{-2}$ | $f^{VI}(x) = -5!(1+x)^{-6}$ | $f''(0) = -1$ | $f^{VI}(0) = -5!$

$f'''(x) = 2!(1+x)^{-3}$ | $f^{VII}(x) = 6!(1+x)^{-7}$ | $f'''(0) = 2!$ | $f^{VII}(0) = 6!$

$$= 0 + \frac{1}{1!}x - \frac{1}{2!}x^2 + \frac{2!}{3!}x^3 - \frac{3!}{4!}x^4 + \frac{4!}{5!}x^5 - \frac{5!}{6!}x^6 + \frac{6!}{7!}x^7$$

$$= 0 + \boxed{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7}}$$

b.) Use your approximation from part a to estimate $\ln(1.1)$ and $\ln(3)$

$\ln(1.1) = \ln(1 + 0.1)$ ← x

$$= 0.1 - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} - \frac{(0.1)^4}{4} + \frac{(0.1)^5}{5} - \frac{(0.1)^6}{6} + \frac{(0.1)^7}{7}$$

$$= \boxed{0.09531018}$$

Good

$$\ln(3) = \ln(1 + 2) = 2 - \frac{(2)^2}{2} + \frac{(2)^3}{3} - \frac{2^4}{4} + \frac{2^5}{5} - \frac{2^6}{6} + \frac{2^7}{7} = \boxed{12.6857}$$

(2 ← x)

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② a.) Graph the 7^{th} and 8^{th} degree Maclaurin polynomials for $f(x) = \ln(1+x)$ together with $f(x)$.

first, $f^{VIII}(x) = -7!(1+x)^{-7}$, $f^{VIII}(0) = -7!$

7^{th} degree Mac laurin polynomial $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7}$

8^{th} degree Maclaurin polynomial $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8}$

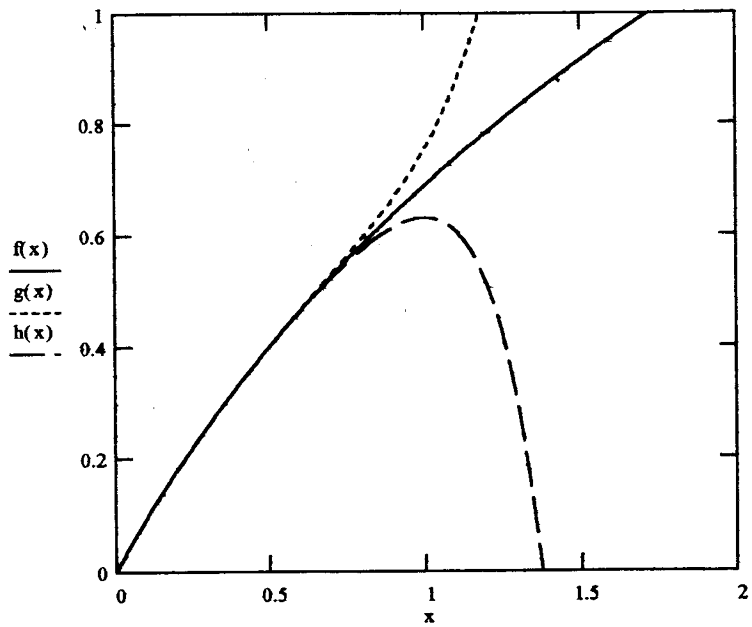
Graph is on computer paper!

(2) cont.

$f(x) := \ln(1+x)$

$g(x) := x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7}$ = 7th degree Maclaurin polynomial

$h(x) := x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8}$ = 8th degree Maclaurin Polynomial



Beautiful

(2) b.) According to my answers, the approximation for $\ln(1.1)$ was very accurate and my approximation for $\ln(3)$ was very inaccurate. This is because for this function, the Maclaurin polynomial works for values of $x = (-1, 1]$. So I can only get an accurate value for values $\ln(0 < x \leq 2)$.

because my function is $\ln(1+x)$

Exactly

c.) Using a higher degree Maclaurin polynomial would not improve the situation. It would make the approximation for $\ln(1.1)$ more accurate, but it would make the approximation for $\ln(3)$ less accurate. To improve the situation, the original function must be changed.

3 a) $x(t) = t^2$, $y(t) = t^3 - ct$

When c is a large negative number the graph stays very close to the y-axis. It is slightly concave down for +y values and slightly concave up for negative y values. As C approaches 0 from negative, it looks more like a big "less than" sign with its point at (0,0). As c is + and small, it looks like the "less than" sign has been shifted to the right and a small loop sits between (0,0) and the "less than" sign. As c grows large and positive the "less than" sign shifts further right and the loop is extended vertically and horizontally.

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b) $\left(\frac{dy}{dx}\right) = \frac{3t^2 - c}{2t}$ horizontal when numerator = 0 $3t^2 - c = 0$

$t = \pm\sqrt{\frac{c}{3}}$

$t = +\sqrt{\frac{c}{3}}$ $x = \frac{c}{3}$ $y = \left(\frac{c}{3}\right)^{3/2} - c\sqrt{\frac{c}{3}}$ $= \left(\frac{c}{3}, \left(\frac{c}{3}\right)^{3/2} - c\sqrt{\frac{c}{3}}\right)$

$t = -\sqrt{\frac{c}{3}}$ $x = \frac{c}{3}$ $y = -\left(\frac{c}{3}\right)^{\frac{3}{2}} + c\sqrt{\frac{c}{3}}$ $= \left(\frac{c}{3}, -\left(\frac{c}{3}\right)^{3/2} + c\sqrt{\frac{c}{3}}\right)$

c) x-intercepts are when $y = 0$ ∴ $t^3 - ct = 0 \rightarrow t(t^2 - c) = 0 \rightarrow t(t - \sqrt{c})(t + \sqrt{c}) = 0$

$= 0$ when $t = (0, -\sqrt{c}, \sqrt{c})$

To find slope of curve plug into $\frac{dy}{dx} = \frac{3t^2 - c}{2t}$

$t = 0$ $\frac{dy}{dx} = \frac{-c}{0}$ → undefined

$t = -\sqrt{c}$ $\frac{dy}{dx} = \frac{3c - c}{-2\sqrt{c}} = -\frac{c\sqrt{c}}{c} = -\sqrt{c}$

$t = +\sqrt{c}$ $\frac{dy}{dx} = \frac{3c - c}{2\sqrt{c}} = \frac{c\sqrt{c}}{c} = \sqrt{c}$

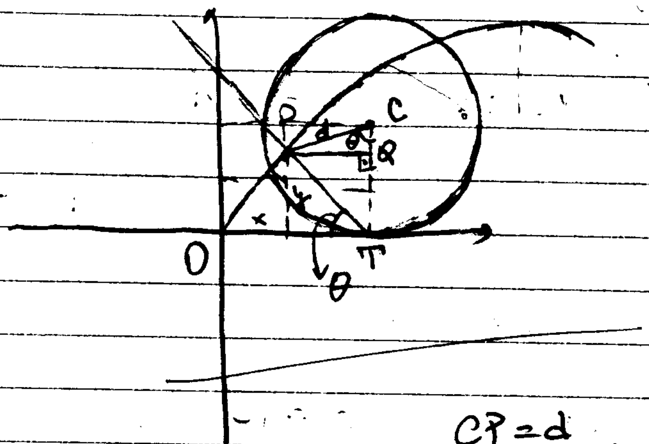
(4) area in loop = $\int_{\alpha}^{b} y(t) \cdot x'(t)\,dt$

5 since it is symetrical we can use $-2\int_0^{\sqrt{c}} (t^2 - ct)(2t)\,dt$

$= -2\int_0^{\sqrt{c}} (2t^4 - 2ct^2)\,dt = -4\int_0^{\sqrt{c}} t^4 - ct^2 = -4\left[\frac{t^5}{5} - \frac{ct^3}{3}\right]_0^{\sqrt{c}}$

$= -4\left[\frac{c^{5/2}}{5} - \frac{c\,c^{3/2}}{3}\right] = -4\left[\frac{3c^{5/2}}{15} - \frac{5c^{5/2}}{15}\right] = \frac{8c^{5/2}}{15}$

(5)



$CP = d$, $CT = r$

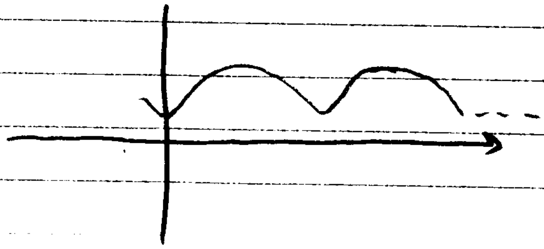
$x = |OT| - |PQ| = r\theta - d\sin\theta$ (from $\triangle QPC$)

$y = |TC| - |QC| = r - d\cos\theta$ (the same $\triangle$)

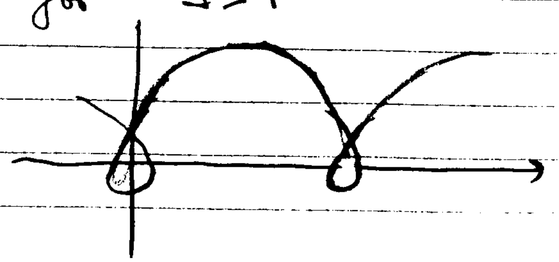
$\Rightarrow$ $\left.\begin{aligned} x &= r\theta - d\sin\theta \\ y &= r - d\cos\theta \end{aligned}\right\}$ are the parametric equ's of the trochoid.

Sketches:

① for $d<r$



for $d>r$



⑥ Length of curve. $x = r\theta - d\sin\theta$
$y = r - d\cos\theta$

$$L = \int_0^{2\pi} \sqrt{\left[(r - d\cos\theta)^2 + d^2\sin^2\theta\right]}\, d\theta$$

$$L = \int_0^{2\pi} \sqrt{r^2 + d^2 - 2rd\cos\theta}$$

(a) $r = 10$ $d = 5$

$$L = \int_0^{2\pi} \sqrt{125 - 100\cos\theta} = 5\int_0^{2\pi} \sqrt{5 - 4\cos\theta} \approx 66.824466$$

(b) $r = 10$ $d = 15$

$$L = \int_0^{2\pi} \sqrt{325 - 300\cos\theta} \approx 105.0502227$$

P.S. It took me a lot of time to try to figure out $\int_0^{2\pi} \sqrt{5 - 4\cos\theta}\, \theta$ without calculator... ☺ Yikes!