# Calculus III Quiz 1 Spring 2000 2/14/2000

1. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{3^n x^n}{n+1}$.

$$\sum_{n=0}^{\infty} \frac{3^n x^n}{n+1}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{3^{n+1} x^{n+1}}{n+2}}{\frac{3^n x^n}{n+1}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{3^n}\cdot 3 \cdot \cancel{x^n} \cdot x \cdot (n+1)}{(n+2)\cdot \cancel{3^n}\,\cancel{x^n}} \right| = \lim_{n\to\infty} \left| 3x \frac{(\cancel{n}+1)}{(\cancel{n}+2)} \right|$$

divide by highest power!

$= 3|x|$ By the ratio test, convergent if $3|x|<1$ $|x| < \frac{1}{3}$ $\boxed{R = \frac{1}{3}}$

Good

2. Determine whether the power series $\sum_{n=0}^{\infty} \frac{3^n x^n}{n+1}$ is convergent or divergent when $x=\frac{1}{3}$.

$$\sum_{n=0}^{\infty} \frac{3^n (\frac{1}{3})^n}{n+1} = \sum_{n=0}^{\infty} \frac{1}{n+1}$$

let $\sum b_n = \sum \frac{1}{n}$ (diverges)

$$\lim_{n\to\infty} \frac{1}{n+1}\left(\frac{n}{1}\right) = \lim_{n\to\infty} \frac{n}{n+1}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1$$

$1>0$ ∴ $a_n$ + $b_n$ diverge together

Divergent

Nice

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3. Give an example of a power series which has a radius of convergence of 7.

Instead of the $3x$ above $= 1/3$, I could get $\frac{x}{7}$.

$$\boxed{\sum_{n=0}^{\infty} \left(\frac{1}{n}\right)\left(\frac{x}{7}\right)^n .}$$

Good thinking.